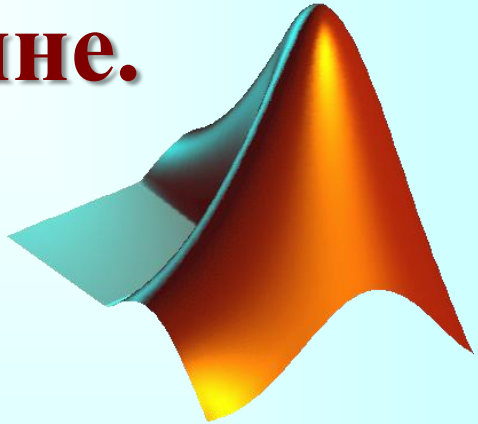
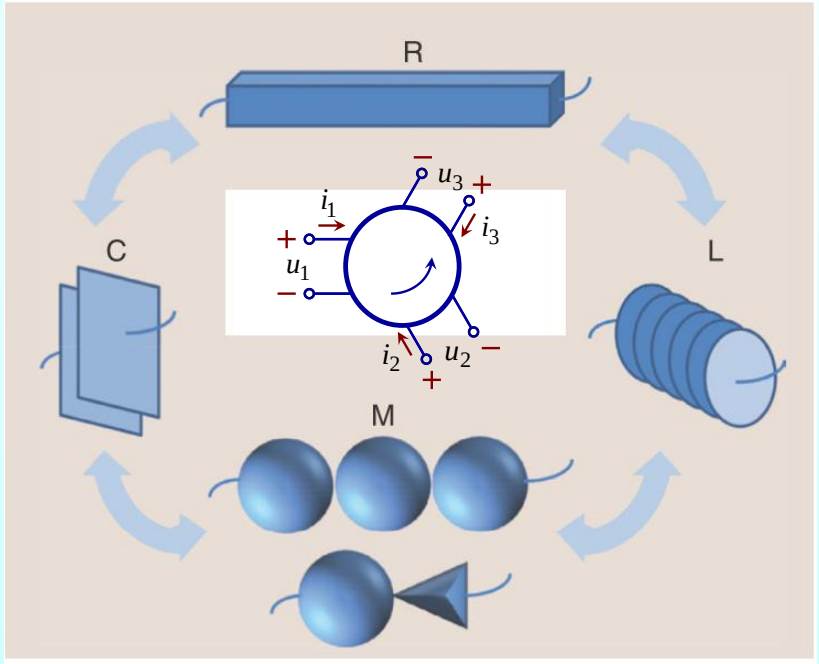
др Милка Потребић Иваниш, ред. проф., др Никола Баста, доцент, 19Е072ПРТК, Универзитет у Београду – Електротехнички факултет, 2024.

Практикум из рачунарске анализе трофазних кола
2. Операциони појачавач. Алгебарске једначине.





Милка Потребић Иваниш

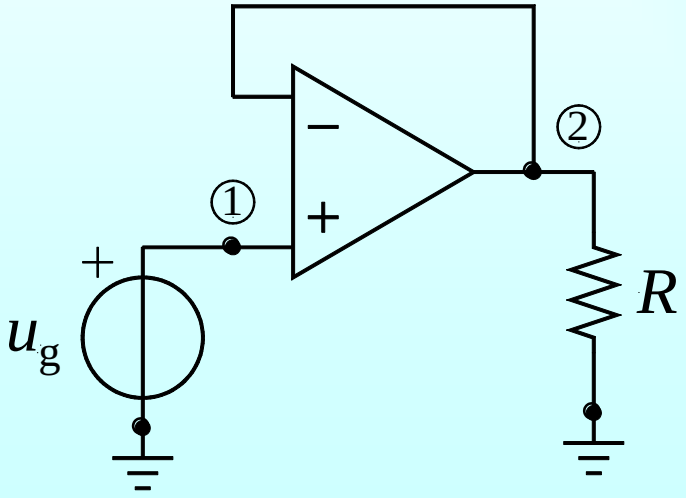
Никола Баста

Рачунарски (софтверски) алати

- *Mathematica*, WolframAlpha, MuPAD, **Maxima**, **SymPy**, **SymPy Live**, **SymPy Gamma**
- MATLAB, **Scilab**, **Octave**, **FreeMat**, **Julia**
- LTspice, **QucsStudio**, **ngspice**, **Xyce**, **XCircuit**
- **Python**, MathCAD, MAPLE, GeoGebra
- Symbolab, SpeQ Mathematics, meta-calculator, Desmos, …

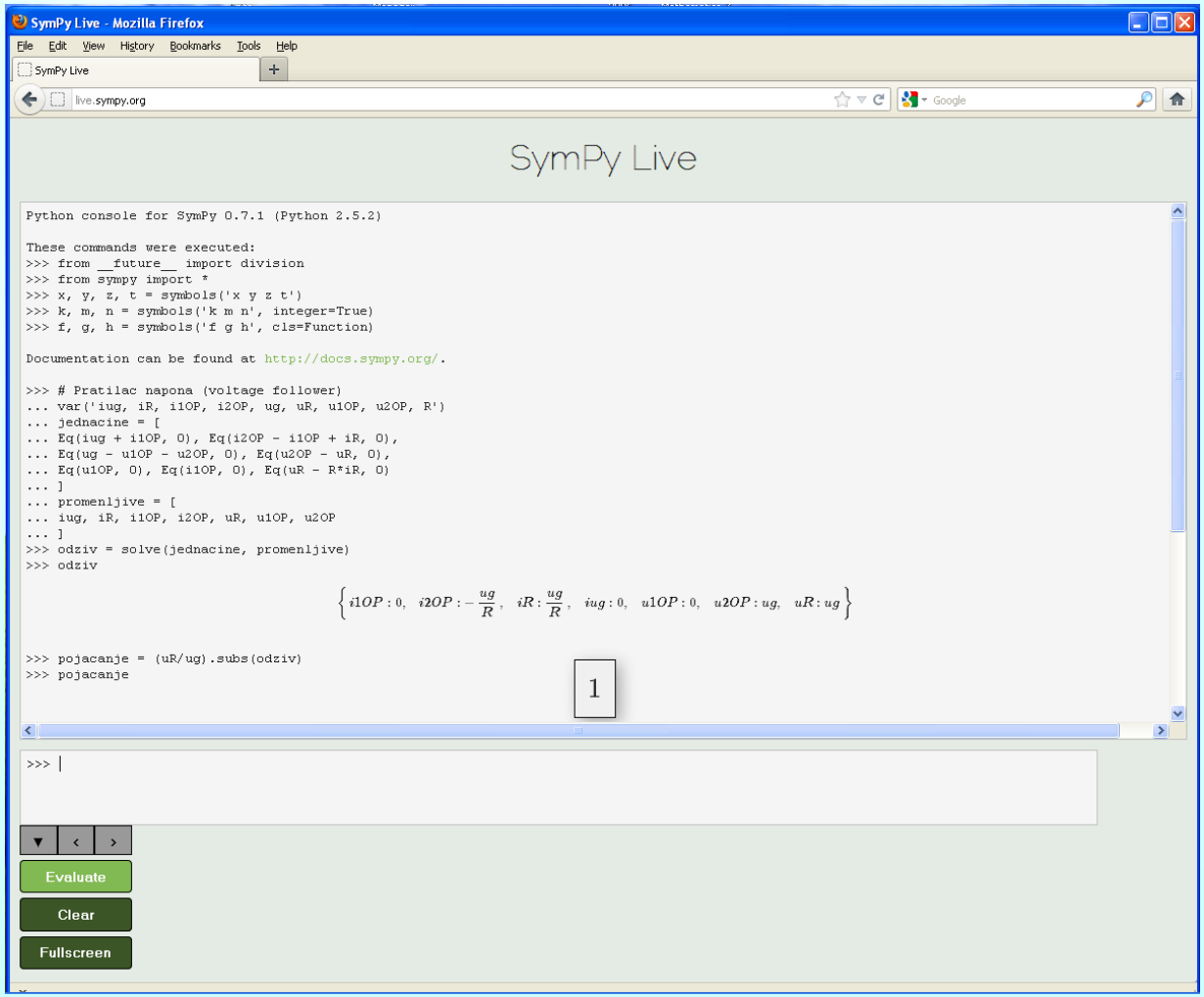
Пратилац напона

Одредити напонско појачање кола са слике, које се назива пратилац напона (voltage follower).



SymPy Live

Symbolic Python Live, http://live.sympy.org/



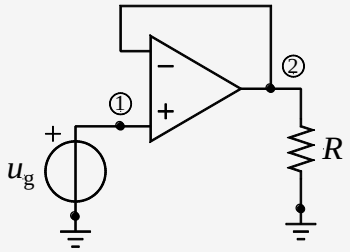
SymPy Live

```
Python console for SymPy 0.7.1 (Python 2.5.2)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)

Documentation can be found at http://docs.sympy.org/.

>>> # Pratilac napona (voltage follower)
... var('iug, iR, i1OP, i2OP, ug, uR, u1OP, u2OP, R')
... jednacine = [
... Eq(iug + i1OP, 0), Eq(i2OP - i1OP + iR, 0),
... Eq(ug - u1OP - u2OP, 0), Eq(u2OP - uR, 0),
... Eq(u1OP, 0), Eq(i1OP, 0), Eq(uR - R*iR, 0)
... ]
... promenljive = [
... iug, iR, i1OP, i2OP, uR, u1OP, u2OP
... ]
>>> odz
>>> odz
```



$$\left\{ i1OP : 0, \quad i2OP : -\frac{ug}{R}, \quad iR : \frac{ug}{R}, \quad iug : 0, \quad u1OP : 0, \quad u2OP : ug, \quad uR : ug \right\}$$

```
>>> pojacanje = (uR/ug).subs(odziv)
>>> pojacanje
```

1

Notepad++, http://notepad-plus-plus.org/

D:\eKola\Zbirka\Alati\a3 Pratilac\PratilacNapona.py - Notepad++

File Edit Search View Encoding Language Settings Macro Run Plugins Window ?

PratilacNapona.py

```
1  # Pratilac napona (voltage follower)
2  var('iug, iR, i1OP, i2OP, ug, uR, u1OP, u2OP, R')
3  jednacine = [
4  Eq(iug + i1OP, 0), Eq(i2OP - i1OP + iR, 0),
5  Eq(ug - u1OP - u2OP, 0), Eq(u2OP - uR, 0),
6  Eq(u1OP, 0), Eq(i1OP, 0), Eq(uR - R*iR, 0)
7  ]
8  promenljive = [
9  iug, iR, i1OP, i2OP, uR, u1OP, u2OP
10 ]
11 odziv = solve(jednacine, promenljive)
12 pojacanje = (uR/ug).subs(odziv)
```

Python file | length : 369 lines : 12 | Ln : 1 Col : 1 Sel : 0 | Dos\Windows | ANSI | INS

MATLAB: Symbolic Math Toolbox

```
clear variables

syms iug iR i1OP i2OP ug uR u1OP u2OP R
promenljive = [iug, iR, i1OP, i2OP, uR, u1OP, u2OP]
```

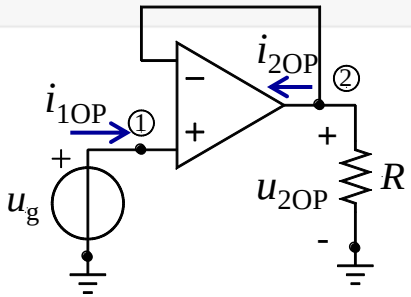
```
promenljive = (iug  iR  i1OP  i2OP  uR  u1OP  u2OP)
```

```
jednacine = [iug + i1OP == 0, i2OP-i1OP+iR == 0, ug-u1OP-u2OP == 0, ...
    u2OP - uR ==0, u1OP == 0, i1OP == 0, uR == R*iR]
```

```
jednacine = (i1OP + iug = 0  i2OP − i1OP + iR = 0  ug − u2OP − u1OP = 0  u2OP − uR = 0  u1OP = 0  i1OP = 0  uR = R iR)
```

```
odziv = solve(jednacine, promenljive)
```

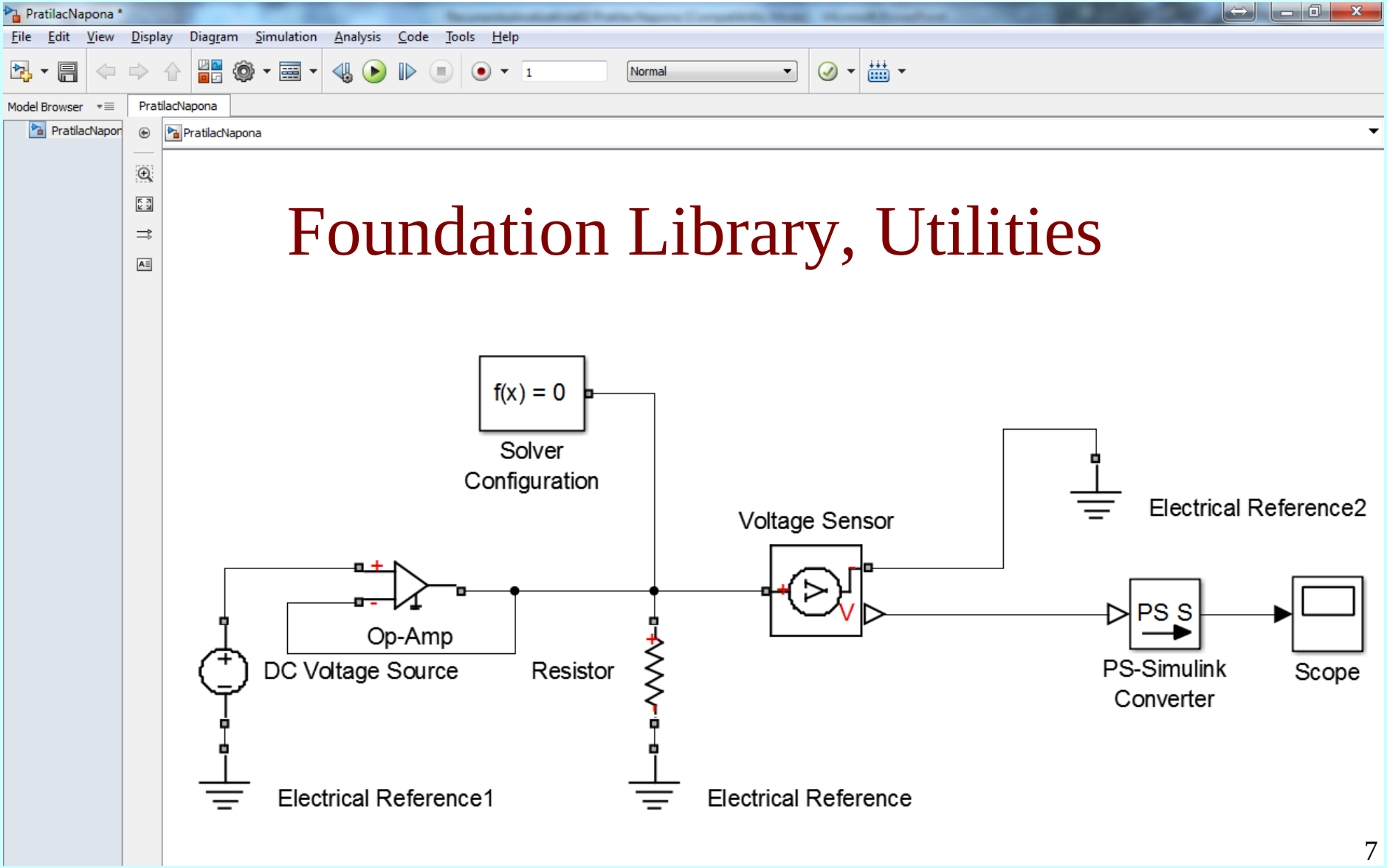
```
odziv = struct with fields:
     iug: 0
      iR: ug/R
    i1OP: 0
    i2OP: -ug/R
      uR: ug
    u1OP: 0
    u2OP: ug
```




```
pojacanje = odziv.u2OP / ug
```

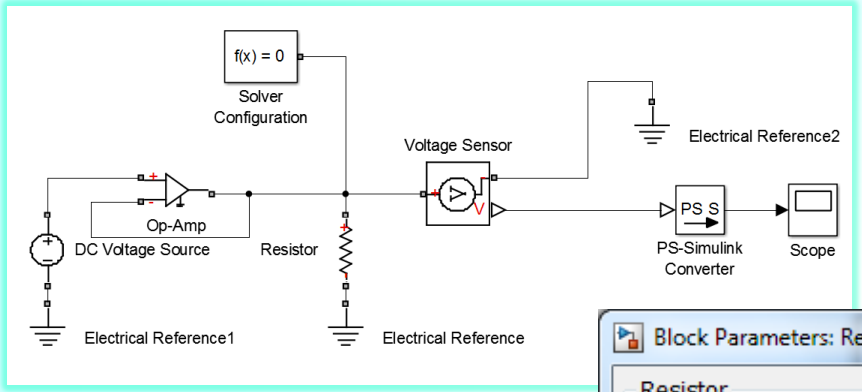
```
pojacanje = 1
```

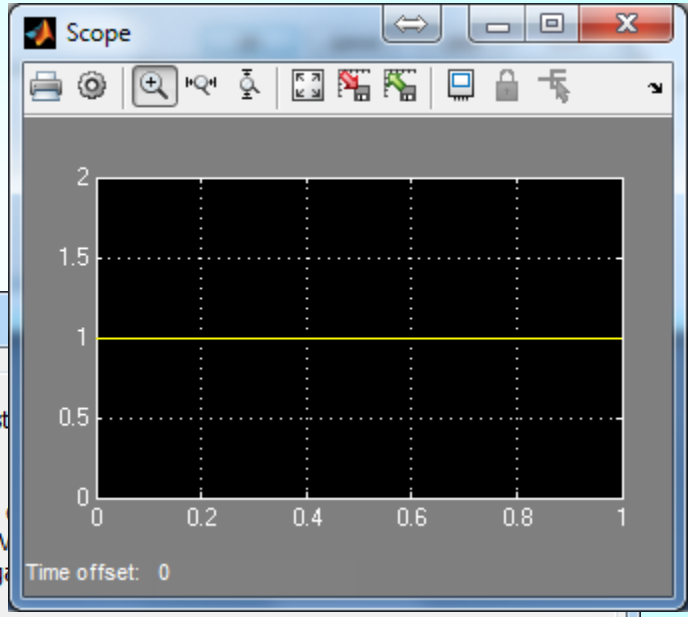
MATLAB: Simscape

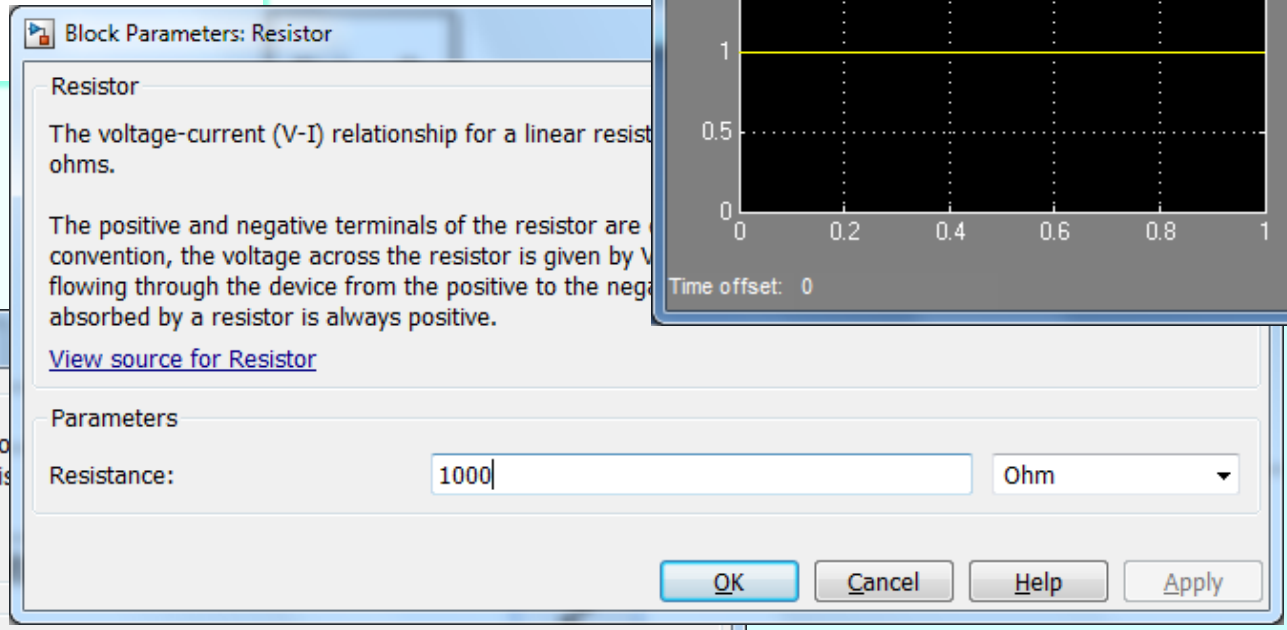


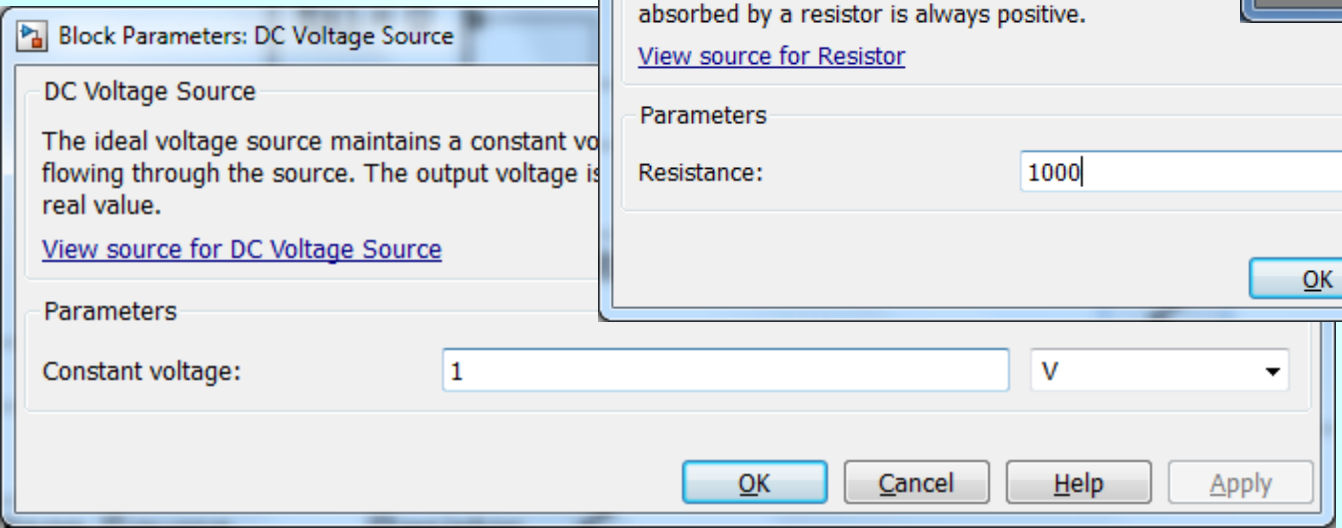
MATLAB: Simscape

Foundation Library, Utilities







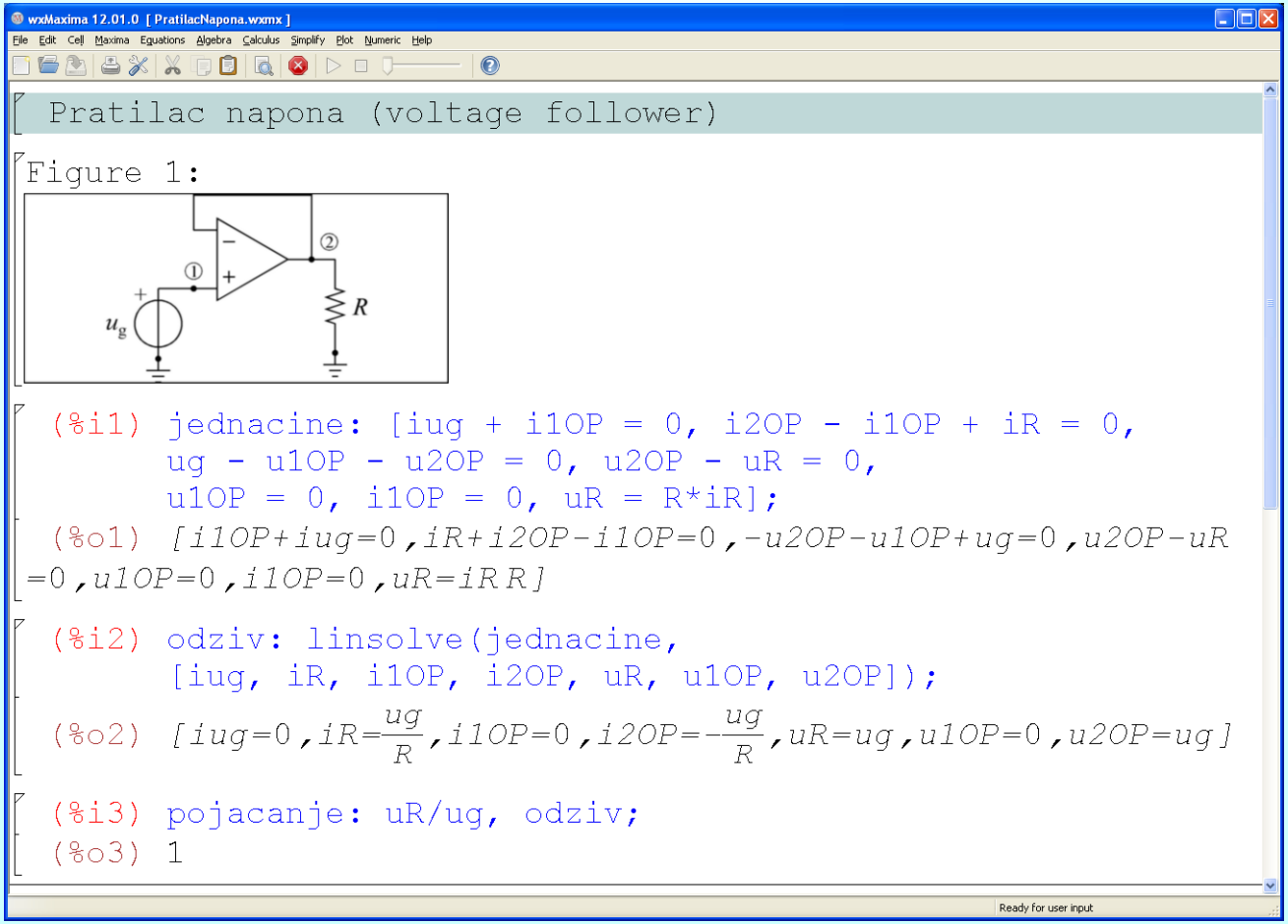


Maxima

Maxima: http://maxima.sourceforge.net/

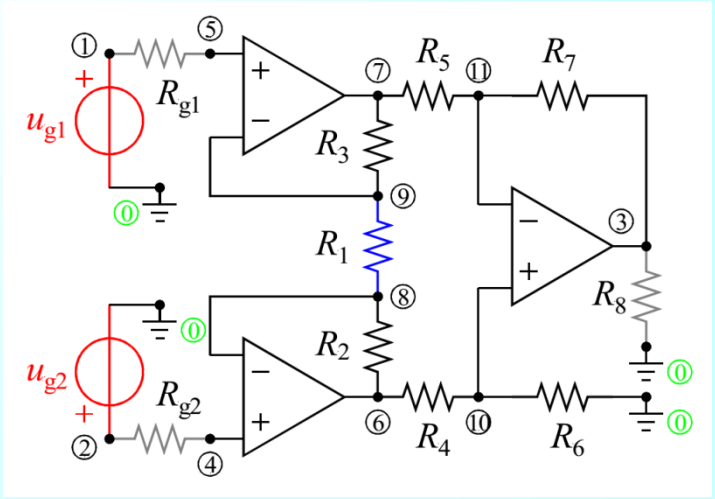
wxMaxima: http://andrejv.github.com/wxmaxima/index.html

CESGA Maxima on line: http://maxima.cesga.es/



Инструментациони појачавач

- Одредити напон v_3 отпорника R_8?
- Нацртати график напона v_3 отпорника R_8 у функцији времена?
- Колика је снага извора?



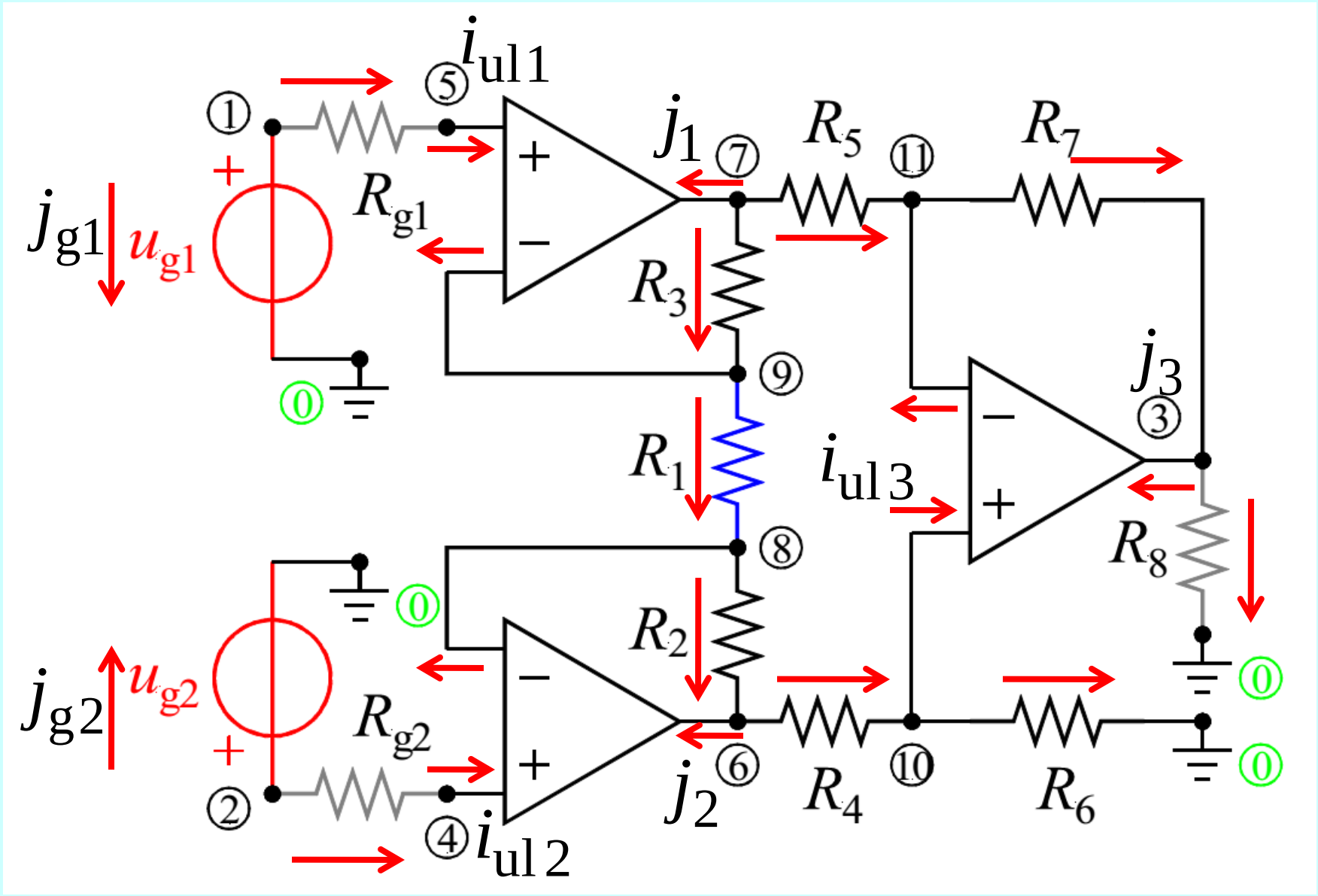
Вредности елемената су познате:

$R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = R$

$R = 1\ \text{k}\Omega,\ R_1 = 2\ \text{k}\Omega,\ R_{g1} = R_{g2} = 1\ \text{k}\Omega$

$u_{g1}(t) = U\,h(t),\ u_{g2}(t) = U\,h(t-T)$

$U = 10\ \text{V},\ T = 1\ \text{s}$


i_{ul1}
j_1
R_5
R_7
j_{g1}
u_{g1}
R_{g1}
R_3
j_3
R_1
i_{ul3}
R_8
R_2
j_{g2}
u_{g2}
R_{g2}
j_2
R_4
R_6
i_{ul2}
①
②
③
④
⑤
⑥
⑦
⑧
⑨
⑩
⑪
⓪

Модификована метода потенцијала чворова

$$(1)\ j_{g1}+\left(V_1-V_5\right)/R_{g1}=0$$

$$(2)\ j_{g2}+\left(V_2-V_4\right)/R_{g2}=0$$

$$(3)\ j_3-\left(V_{11}-V_3\right)/R_7+V_3/R_8=0$$

$$(4)-\left(V_2-V_4\right)/R_{g2}+i_{ul2}=0$$

$$(5)-\left(V_1-V_5\right)/R_{g1}+i_{ul1}=0$$

$$(6)\ j_2+\left(V_6-V_{10}\right)/R_4-\left(V_8-V_6\right)/R_2=0$$

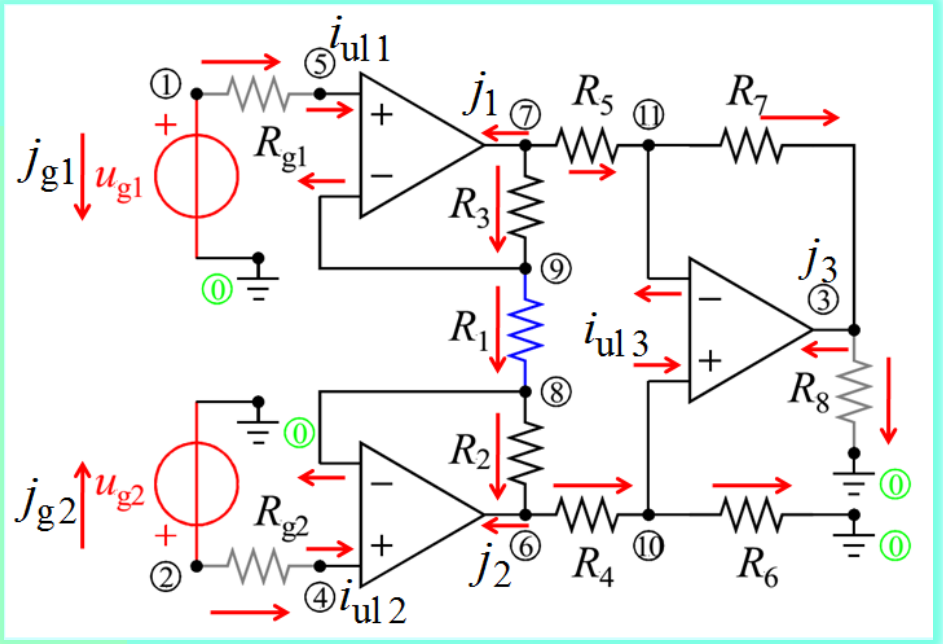
$$(7)\ j_1+\left(V_7-V_9\right)/R_3+\left(V_7-V_{11}\right)/R_5=0$$

$$(8)-i_{ul2}-\left(V_9-V_8\right)/R_1+\left(V_8-V_6\right)/R_2=0$$

$$(9)-i_{ul1}-\left(V_7-V_9\right)/R_3+\left(V_9-V_8\right)/R_1=0$$

$$(10)-\left(V_6-V_{10}\right)/R_4+i_{ul3}+V_{10}/R_6=0$$

$$(11)-\left(V_7-V_{11}\right)/R_5-i_{ul3}+\left(V_{11}-V_3\right)/R_7=0$$



$$i_{ul1}=0$$

$$i_{ul2}=0$$

$$i_{ul3}=0$$

$$V_5=V_9$$

$$V_4=V_8$$

$$V_{10}=V_{11}$$

$$V_1=u_{g1}$$

$$V_2=u_{g2}$$

MATLAB: Symbolic Math Toolbox

Промењљиве, једначине, решавање

```
clear variables

syms R R1 R2 R3 R4 R5 R6 R7 R8 Rg1 Rg2 iul1 iul2 iul3 j1 j2 j3 jg1 jg2 ug1 ug2 v1 v10 v11 v2 v3 v4 v5 v6 v7 v8 v9
jednacine = [jg1 + (v1 - v5)/Rg1 == 0,...
    v1 == ug1, jg2 + (v2 - v4)/Rg2 == 0,...
    v2 == ug2, j3 + v3/R8 - (v11 - v3)/R7 == 0,...
    -((v2 - v4)/Rg2) + iul2 == 0, -((v1 - v5)/Rg1) + iul1 == 0,...
    j2 + (v6 - v10)/R4 - (v8 - v6)/R2 == 0, j1 + (v7 - v11)/R5 + (v7 - v9)/R3 == 0,...
    -iul2 - (v9 - v8)/R1 + (v8 - v6)/R2 == 0, -iul1 - (v7 - v9)/R3 + (v9 - v8)/R1 == 0,...
    -((v6 - v10)/R4) + v10/R6 + iul3 == 0, -((v7 - v11)/R5) + (v11 - v3)/R7 - iul3 == 0,...
    iul1 == 0, iul2 == 0, iul3 == 0, ...
    v5 == v9, v4 == v8, v10 == v11]
```

syms

sym

```
jednacine =
```

$$\left(\mathrm{jg}_1+\frac{v_1-v_5}{\mathrm{Rg}_1}=0\quad v_1=\mathrm{ug}_1\quad \mathrm{jg}_2+\frac{v_2-v_4}{\mathrm{Rg}_2}=0\quad v_2=\mathrm{ug}_2\quad j_3+\frac{v_3}{R_8}+\frac{v_3-v_{11}}{R_7}=0\quad \mathrm{iul}_2-\frac{v_2-v_4}{\mathrm{Rg}_2}=0\quad \mathrm{iul}_1-\frac{v_1-v_5}{\mathrm{Rg}_1}=0\quad j_2+\frac{v_6-v_8}{R_2}+\frac{v_6-v_{10}}{R_4}=\right.$$

assume

```
assume(0 < Rg1 & 0 < Rg2 & 0 < R1 & 0 < R2 & 0 < R3 & 0 < R4 & 0 < R5 & 0 < R6 & 0 < R7 & 0 < R8 & 0 < R)

promenljive = [v1, v2, v3, v4, v5, v6, v7, v8, v9, v10, v11, j1, j2, j3, iul1, iul2, iul3, jg1, jg2]
```

$$\mathrm{promenljive} = \left(v_1\ \ v_2\ \ v_3\ \ v_4\ \ v_5\ \ v_6\ \ v_7\ \ v_8\ \ v_9\ \ v_{10}\ \ v_{11}\ \ j_1\ \ j_2\ \ j_3\ \ \mathrm{iul}_1\ \ \mathrm{iul}_2\ \ \mathrm{iul}_3\ \ \mathrm{jg}_1\ \ \mathrm{jg}_2\right)$$

solve

```
odziv = solve(jednacine, promenljive)
```

```
odziv = struct with fields:
      v1: ug1
      v2: ug2
      v3: -(R1*R4*R7*ug1 - R1*R5*R6*ug2 + R2*R5*R6*ug1 + R1*R6*R7*ug1 - R2*R5*R6*ug2 + R3*R4*R7
      v4: ug2
```

Замена променљивих и вредности

```
zamena = [R2 == R, R3 == R, R4 == R, R5 == R, R6 == R, R7 == R]
```

zamena = $\left(R_2 = R \quad R_3 = R \quad R_4 = R \quad R_5 = R \quad R_6 = R \quad R_7 = R\right)$

```
odziv.v3
```

ans =

$$-\frac{R_1 R_4 R_7 \mathrm{ug}_1 - R_1 R_5 R_6 \mathrm{ug}_2 + R_2 R_5 R_6 \mathrm{ug}_1 + R_1 R_6 R_7 \mathrm{ug}_1 - R_2 R_5 R_6 \mathrm{ug}_2 + R_3 R_4 R_7 \mathrm{ug}_1 - R_1 R_6 R_7 \mathrm{ug}_2 + R_2 R_6 R_7 \mathrm{ug}_1 - R_3 R_4 R_7 \mathrm{ug}_2 - R_2 R_6 R_7 \mathrm{ug}_2 + R_3 R_6 R_7 \mathrm{ug}_1 - R_3 R_6 R_7 \mathrm{ug}_2}{R_1 R_5 \left(R_4 + R_6\right)}$$

```
subs(odziv.v3, lhs(zamena), rhs(zamena))
```

subs

ans =

$$-\frac{4 R^3 \mathrm{ug}_1 - 4 R^3 \mathrm{ug}_2 + 2 R^2 R_1 \mathrm{ug}_1 - 2 R^2 R_1 \mathrm{ug}_2}{2 R^2 R_1}$$

simplify

```
v3Sol = simplify(subs(odziv.v3, lhs(zamena), rhs(zamena)))
```

v3Sol =

$$-\frac{\left(2 R + R_1\right)\left(\mathrm{ug}_1 - \mathrm{ug}_2\right)}{R_1}$$

MATLAB: Symbolic Math Toolbox

График решења

```
syms T U t uR8(t) ug1(t) ug2(t)
assume(0 < U & 0 < T & 0 < t);
pobude = [ug1 == U * heaviside(t-T), ug2 == U * heaviside(t - 2*T)]
```

pobude(t) = $\left(\mathrm{ug}_1(t) = U\ \mathrm{heaviside}(t-T) \quad \mathrm{ug}_2(t) = U\ \mathrm{heaviside}(t-2T)\right)$

```
subs(v3Sol, lhs(pobude), rhs(pobude))
```

ans =

$$-\frac{(2R+R_1)\ (U\ \mathrm{heaviside}(t-T) - U\ \mathrm{heaviside}(t-2T))}{R_1}$$

```
vrednosti = [U == sym(10), T == sym(1), R == sym(1000), R1 == sym(2000)]
```

vrednosti = $\left(U = 10 \quad T = 1 \quad R = 1000 \quad R_1 = 2000\right)$

```
uR8(t) = subs(subs(v3Sol, lhs(pobude), rhs(pobude)), lhs(vrednosti), rhs(vrednosti))
```

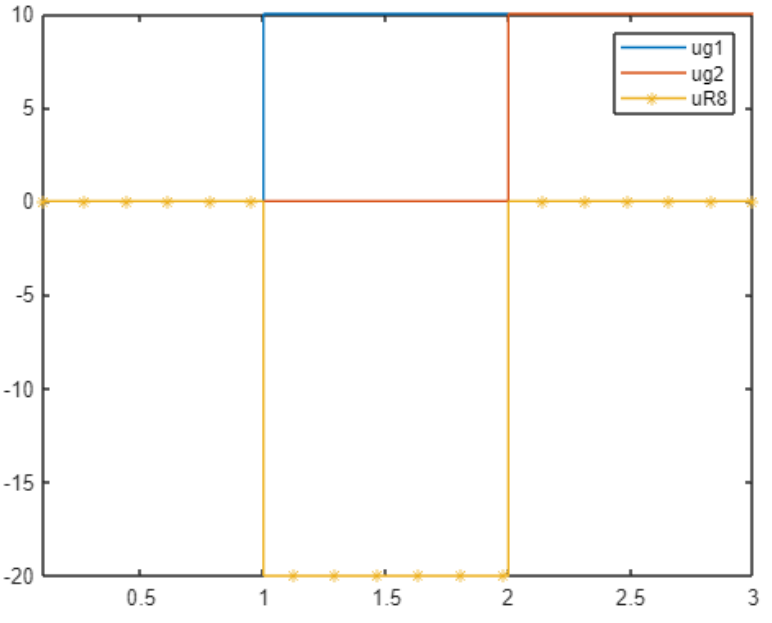
uR8(t) = $20\ \mathrm{heaviside}(t-2) - 20\ \mathrm{heaviside}(t-1)$

```
ug1(t) = subs(subs(ug1, lhs(pobude), rhs(pobude)), lhs(vrednosti), rhs(vrednosti))
```

ug1(t) = $10\ \mathrm{heaviside}(t-1)$

```
ug2(t) = subs(subs(ug2, lhs(pobude), rhs(pobude)), lhs(vrednosti), rhs(vrednosti))
```

ug2(t) = $10\ \mathrm{heaviside}(t-2)$




```
figure
fplot(ug1(t), [0.1, 3])
hold on
fplot(ug2(t), [0.1, 3])
fplot(uR8(t), [0.1, 3], '-*')
hold off
legend({'ug1', 'ug2', 'uR8'});
```

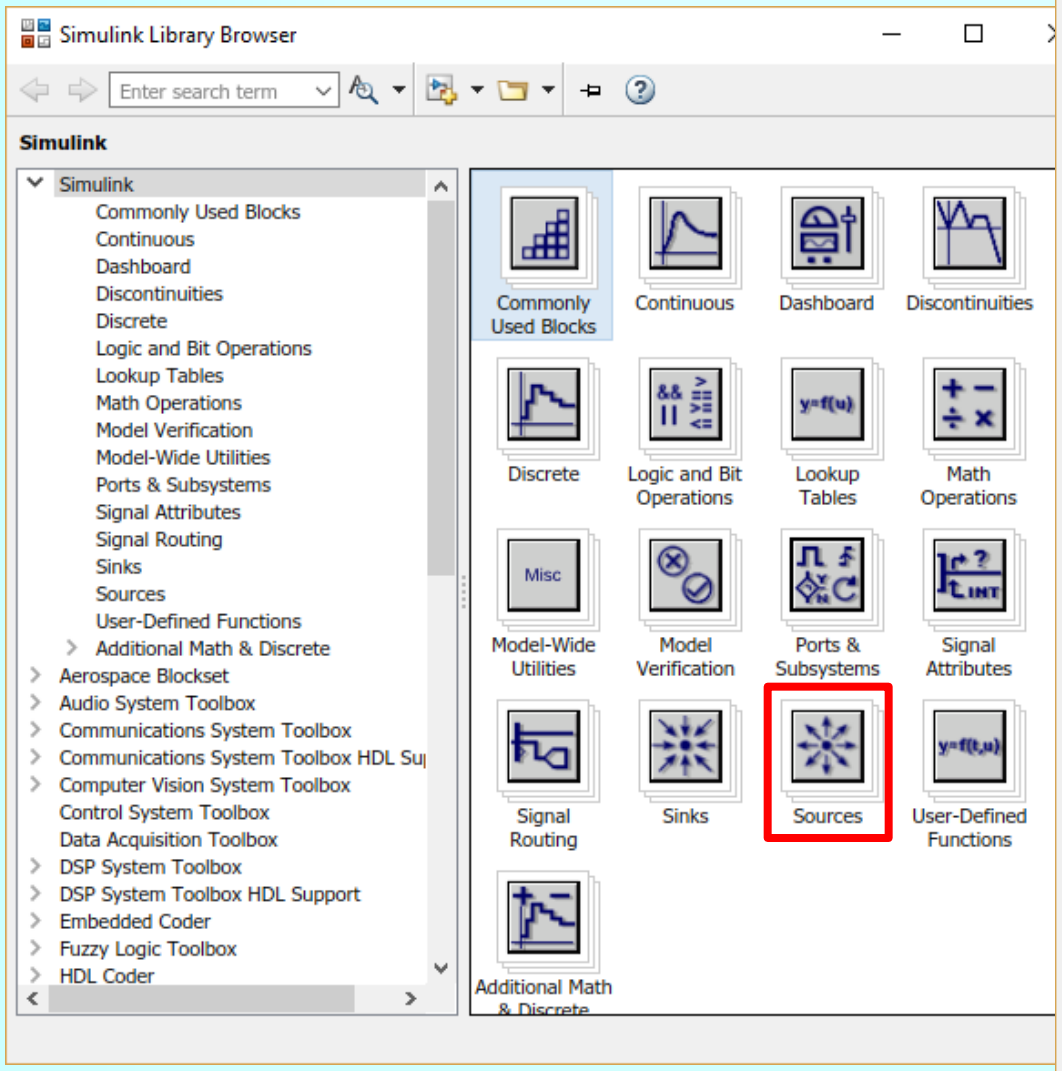
figure
fplot

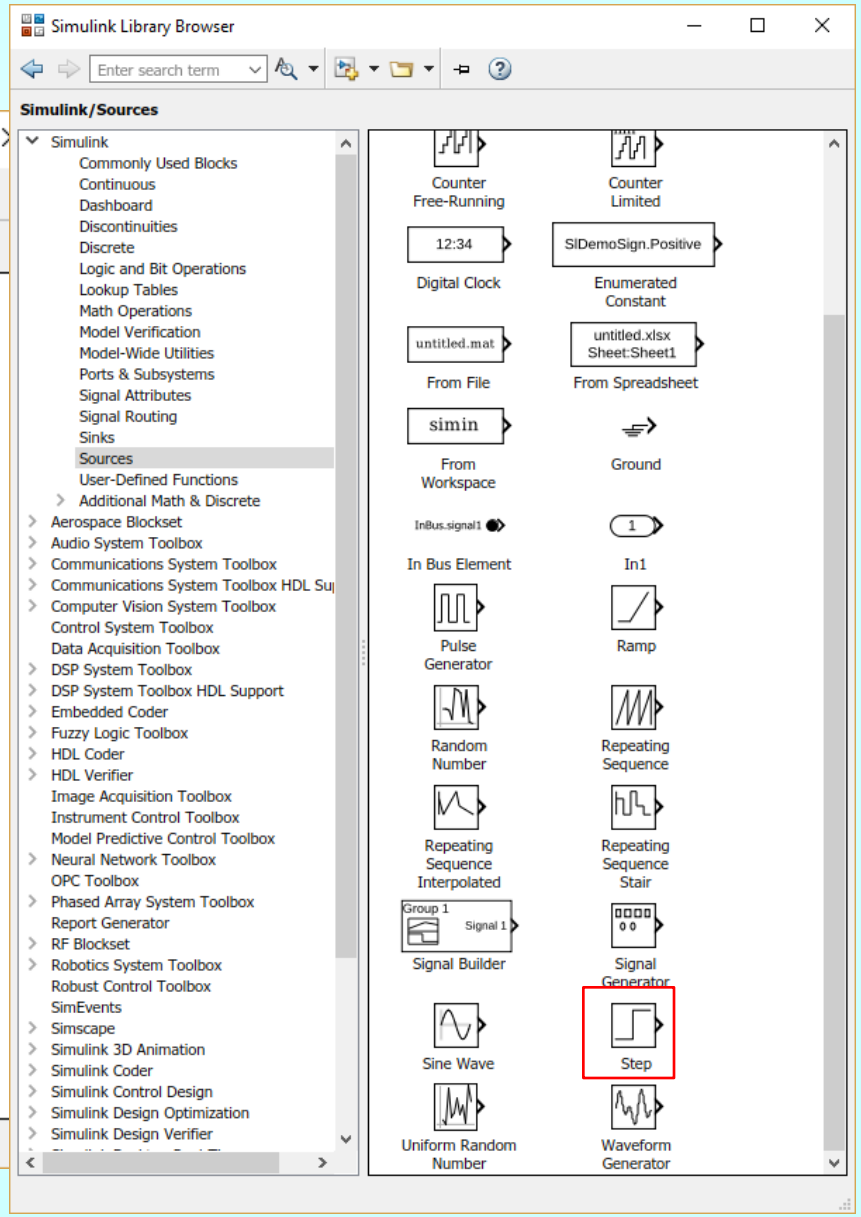
```
pg = subs(jg1*ug1 + jg2*ug2, odziv)
```

pg(t) = 0

MATLAB: Simulink

Sources

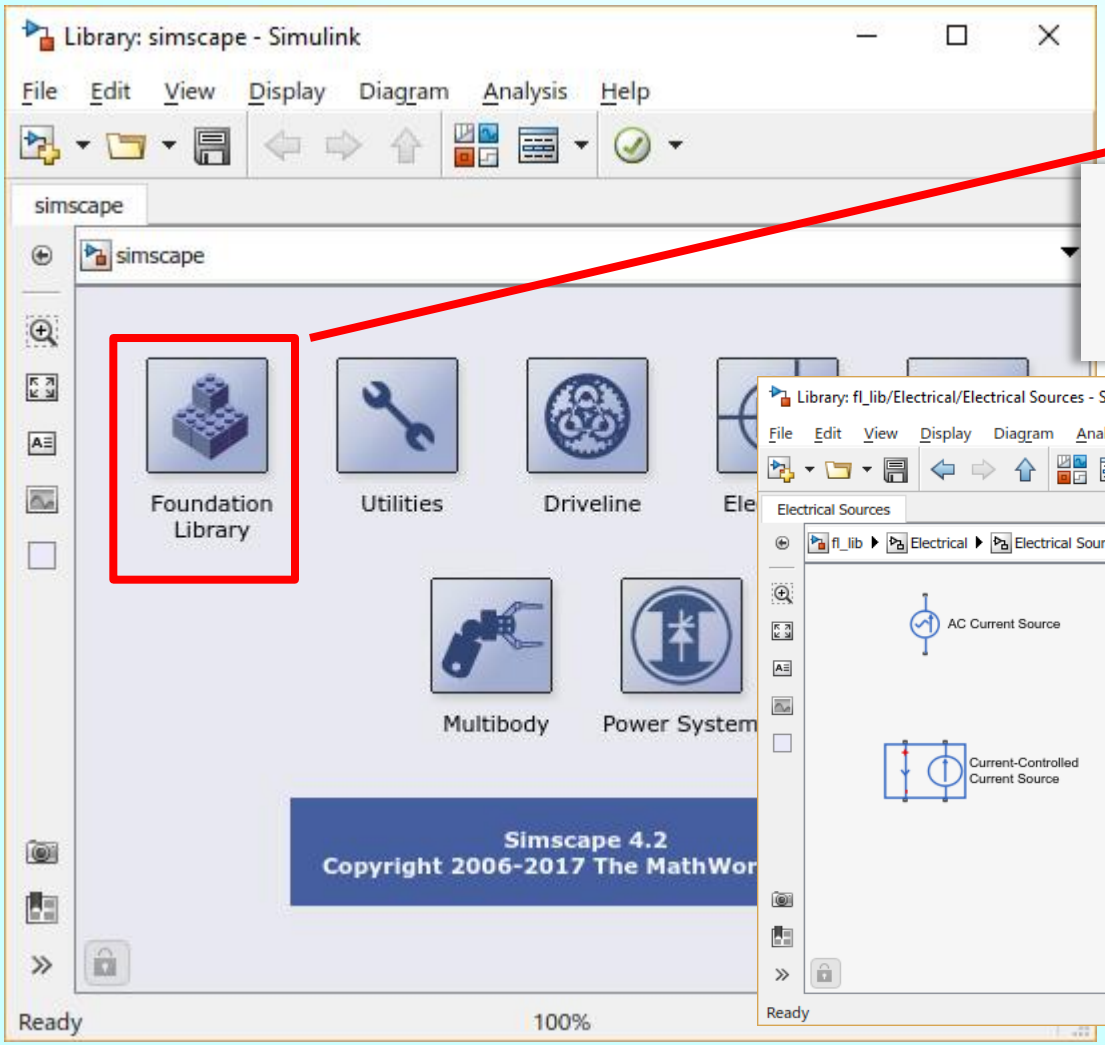


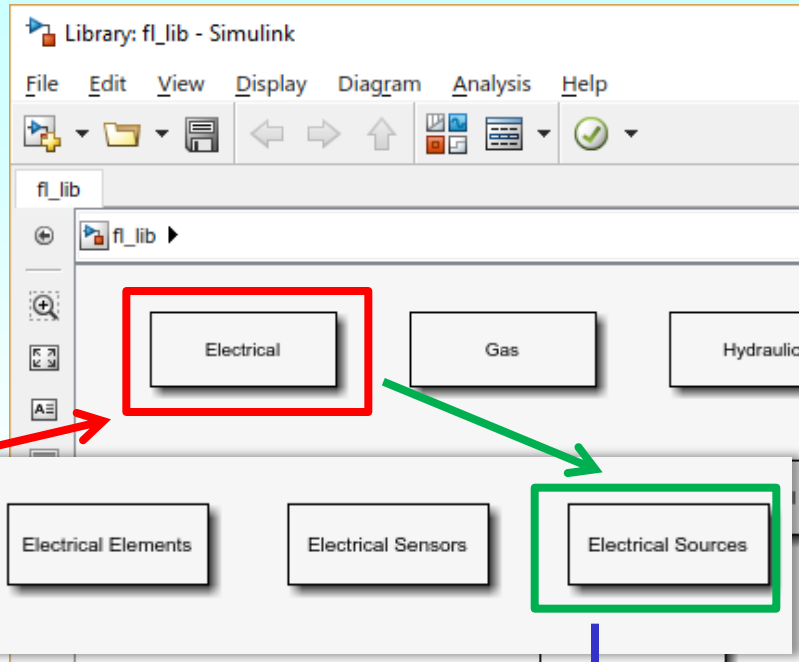


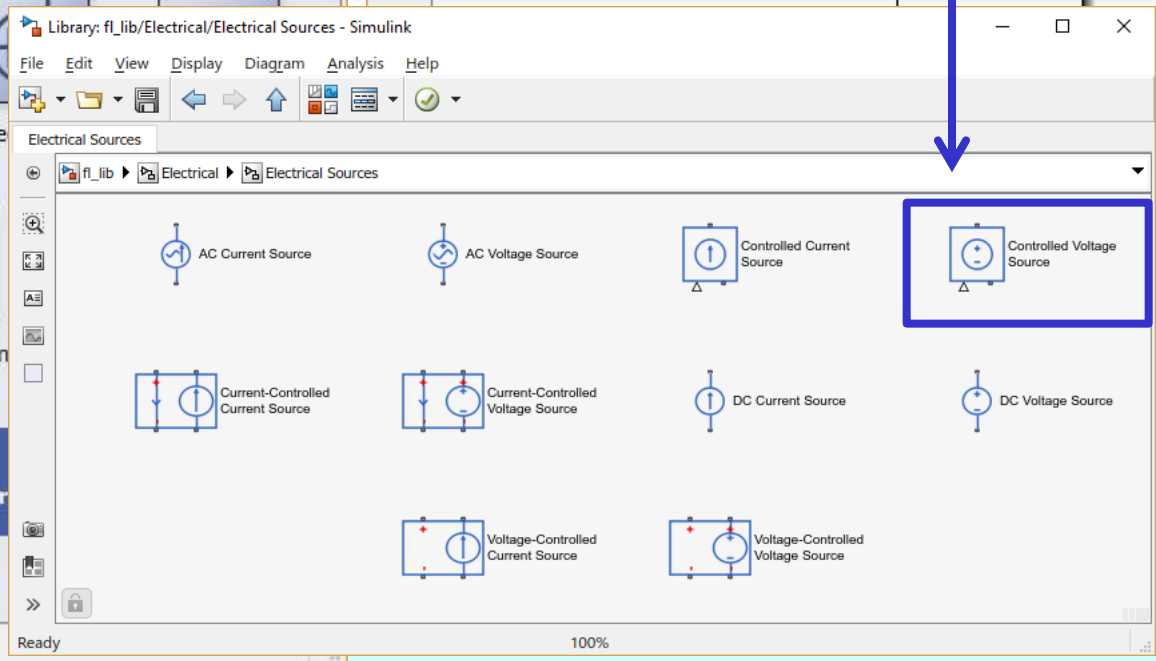
MATLAB: Simscape

Foundation Library

Electrical, Electrical Sources

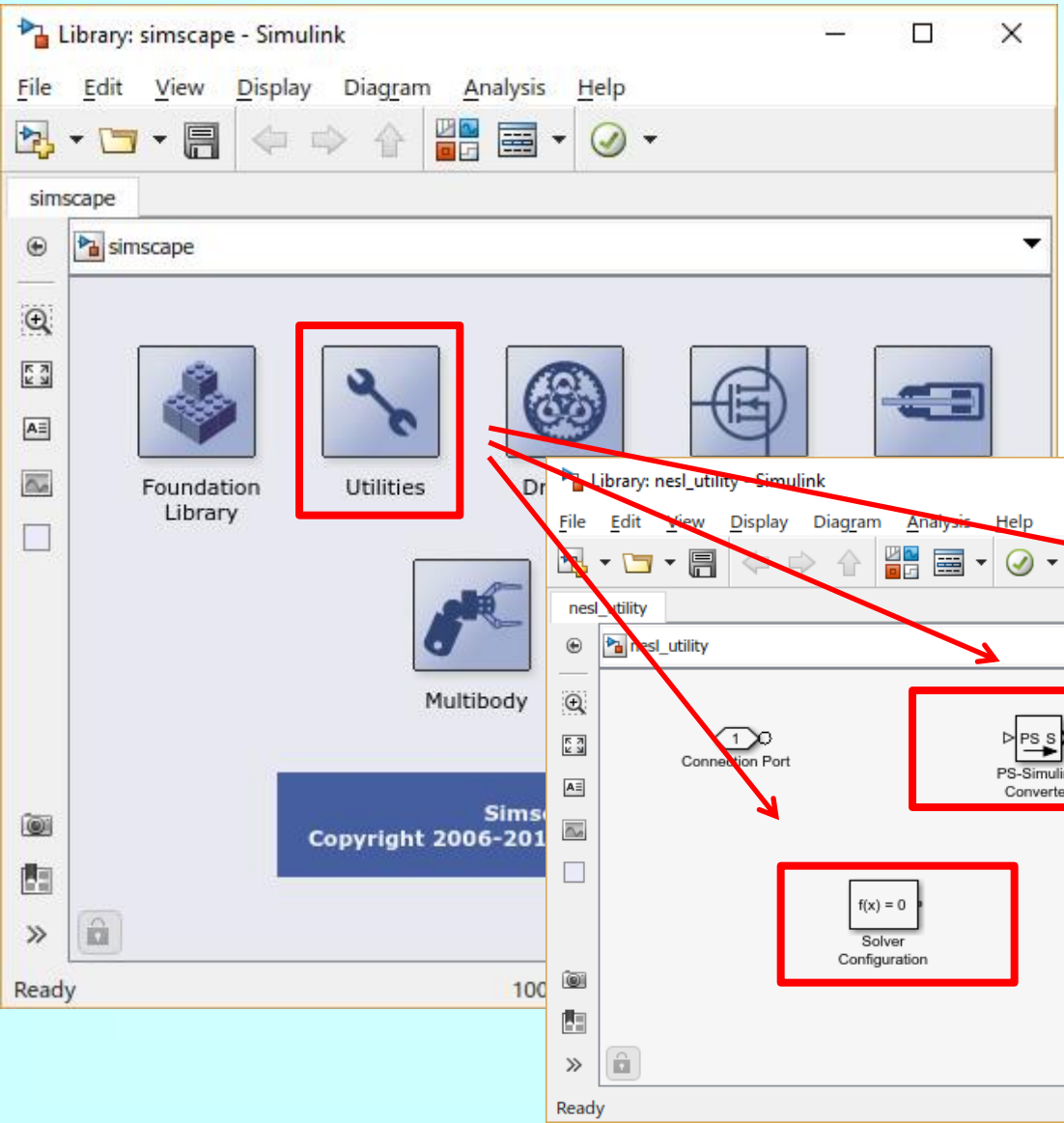


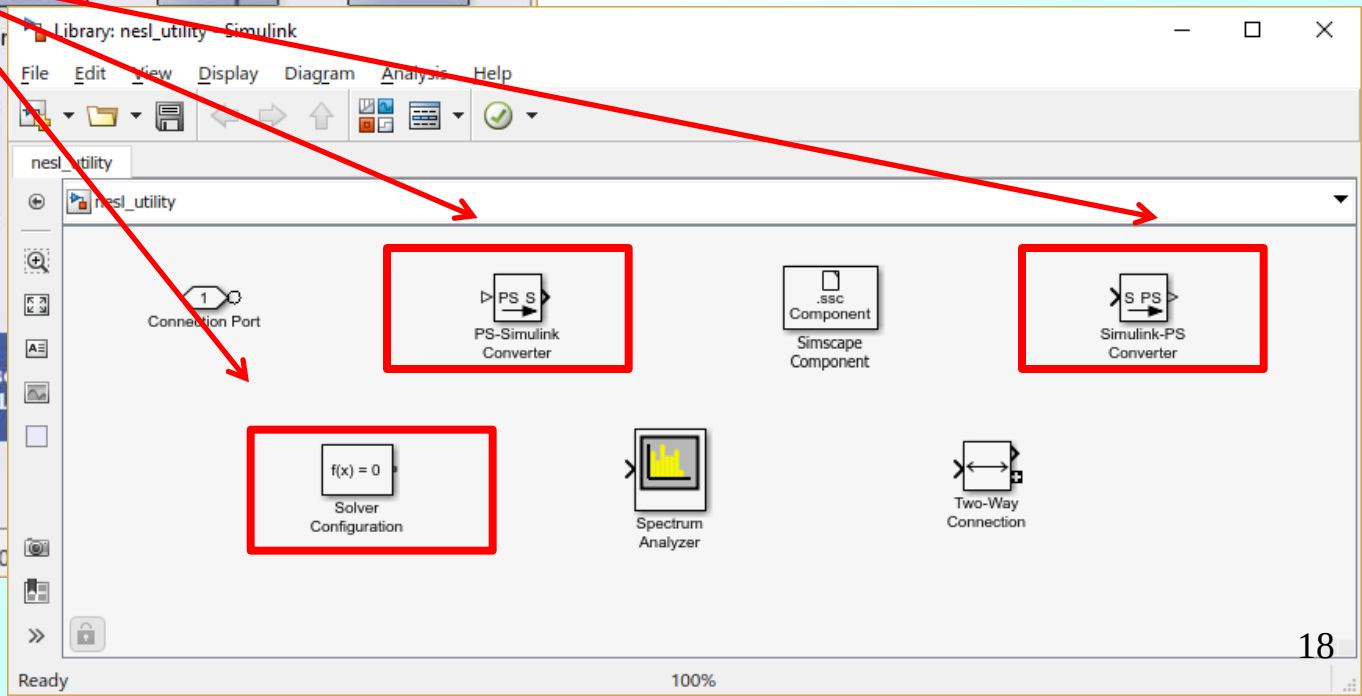




MATLAB: Simscape

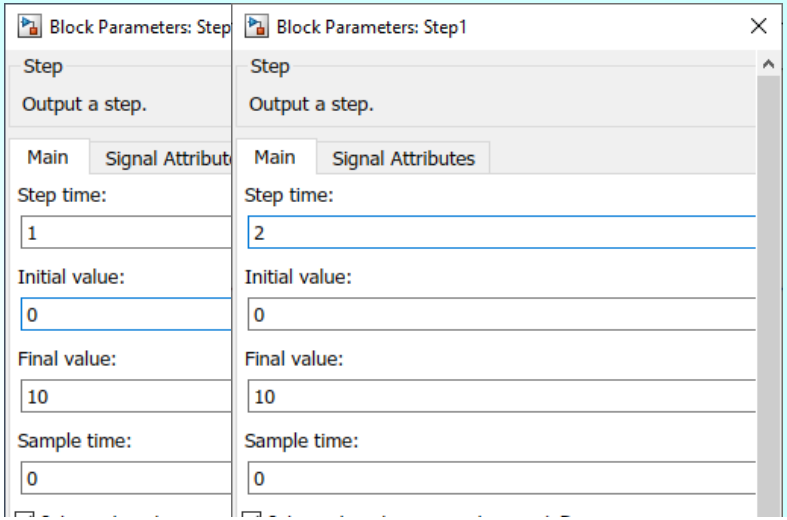
Utilities

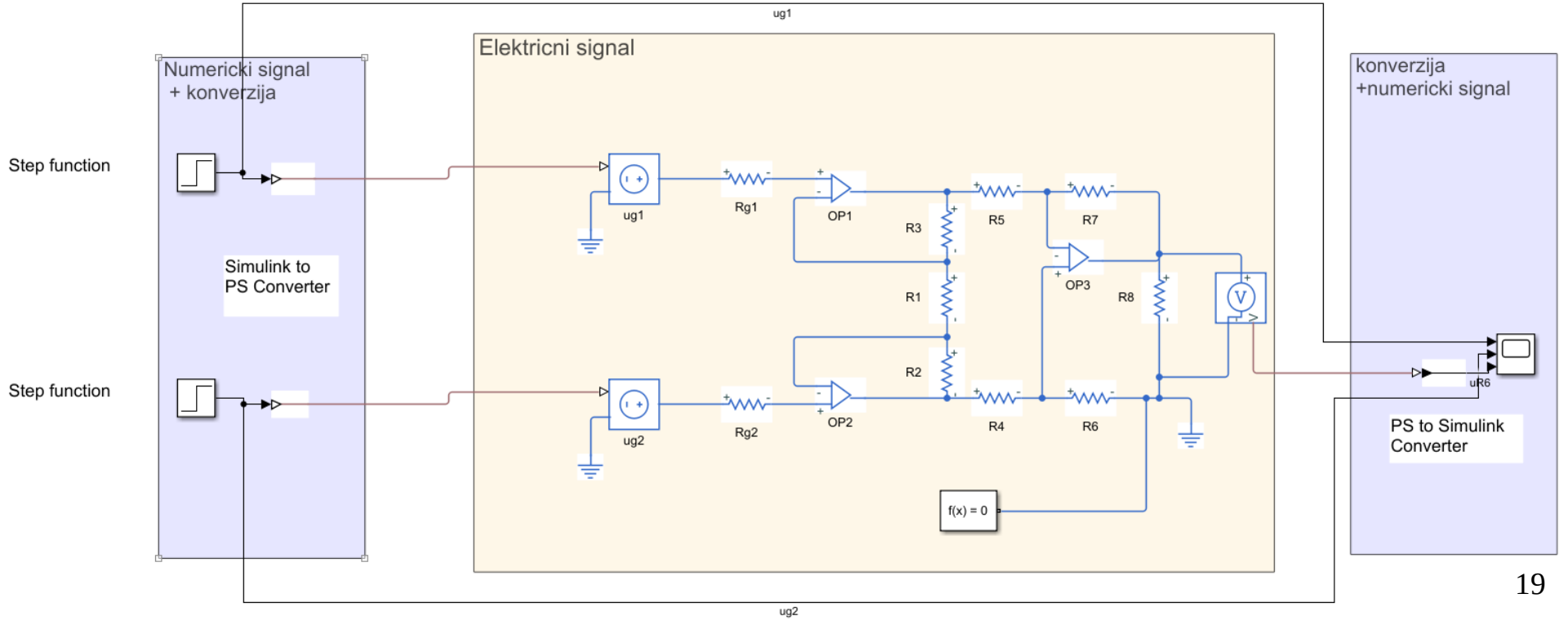


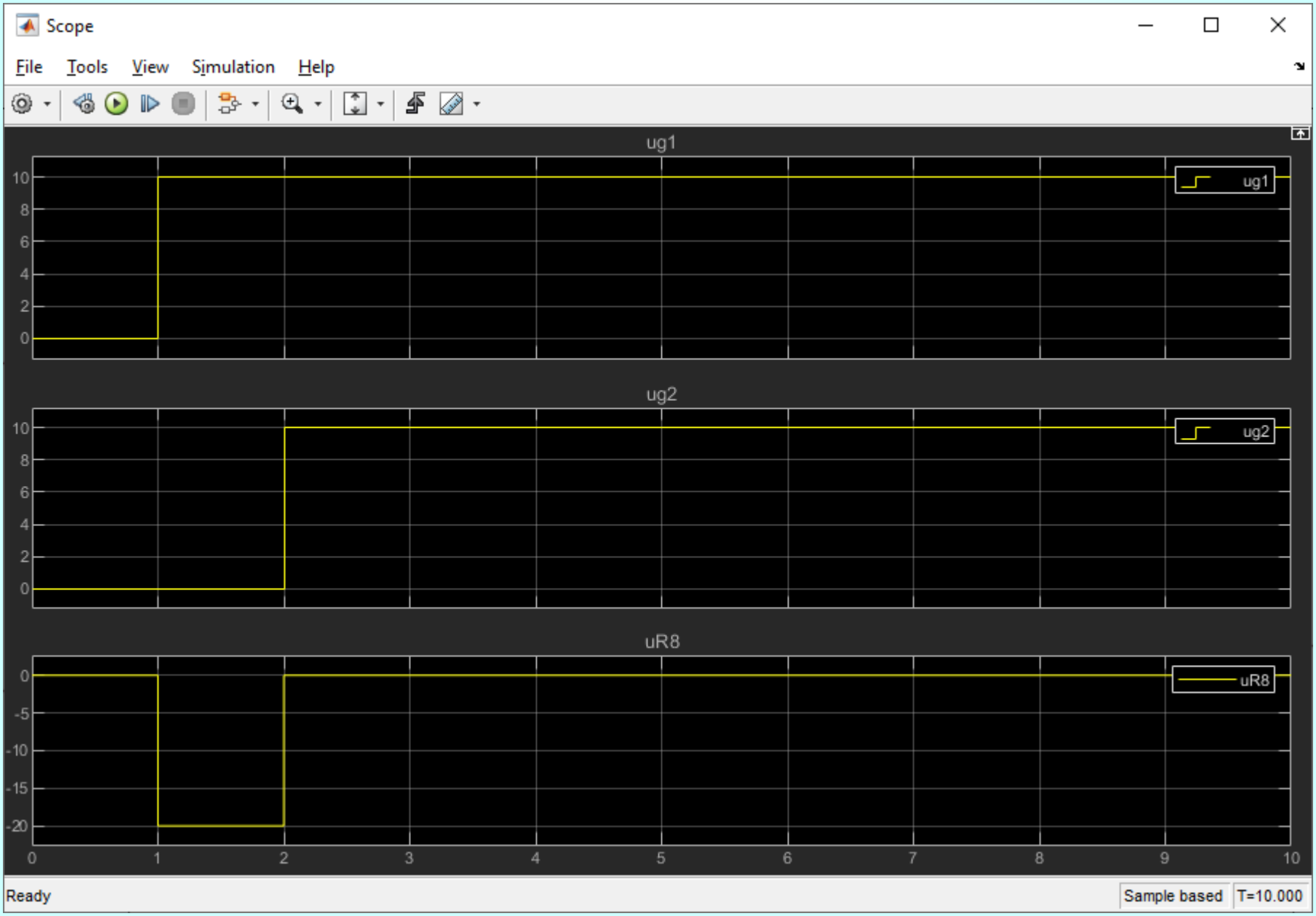


MATLAB: Simscape

Foundation Library, Utilities, Simulink

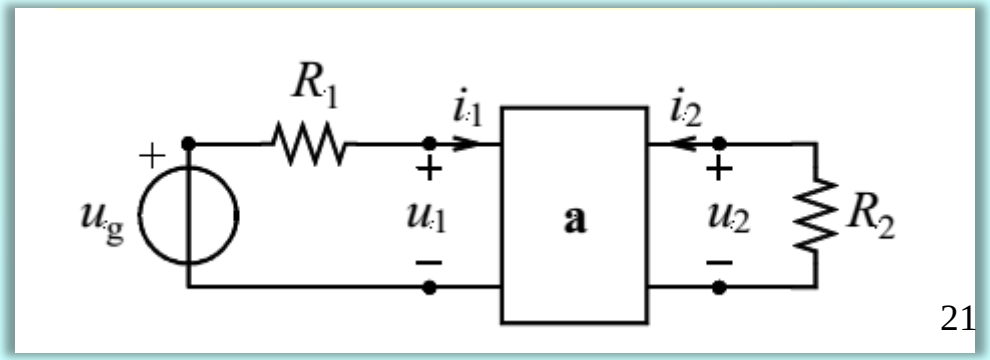





Scope
File Tools View Simulation Help
ug1
ug1
10
8
6
4
2
0
ug2
ug2
10
8
6
4
2
0
uR8
uR8
0
-5
-10
-15
-20
0 1 2 3 4 5 6 7 8 9 10
Ready
Sample based
T=10.000

ABCD параметри мреже

- Отпорности отпорника и *a*-параметри (*ABCD*-параметри) мреже са два приступа су познати.
- Одредити:
  - Количник напона $A = u_2 / u_g$
  - Улазну отпорност мреже $R_u = u_1/i_1$
  - Струју i_1
- Сматрати да су *a*-параметри мреже реални и различити од нуле.



```
clear all

syms R1 R2 a11 a12 a21 a22 i1 i2 u1 u2 ug
jednacine = [ug == R1 * i1 + u1, u2 + R2 * i2 == 0, u1 == a11 * u2 + a12 * -i2, i1 == a21 * u2 + a22 * -i2]
```

jednacine = $\left(\mathrm{ug} = u_1 + R_1\, i_1 \quad u_2 + R_2\, i_2 = 0 \quad u_1 = a_{11}\, u_2 - a_{12}\, i_2 \quad i_1 = a_{21}\, u_2 - a_{22}\, i_2\right)$

```
promenljive = [u1, u2, i1, i2]
```

promenljive = $\left(i_1 \quad i_2 \quad u_1 \quad u_2\right)$

```
assume(a11*a22-a12*a21 ~= 0 & 0<R1 & 0<R2 & ug ~= 0);

odziv = solve(jednacine, promenljive)
```

```
odziv = struct with fields:
    i1: (a22*ug + R2*a21*ug)/(a12 + R2*a11 + R1*a22 + R1*R2*a21)
    i2: -ug/(a12 + R2*a11 + R1*a22 + R1*R2*a21)
    u1: (ug*(a12 + R2*a11))/(a12 + R2*a11 + R1*a22 + R1*R2*a21)
    u2: (R2*ug)/(a12 + R2*a11 + R1*a22 + R1*R2*a21)
```

```
odziv.i2
```

ans =

$$-\frac{\mathrm{ug}}{a_{12} + R_2\, a_{11} + R_1\, a_{22} + R_1\, R_2\, a_{21}}$$

```
A = odziv.u2 / ug
```

A =

$$\frac{R_2}{a_{12} + R_2\, a_{11} + R_1\, a_{22} + R_1\, R_2\, a_{21}}$$

MATLAB: Symbolic Math Toolbox