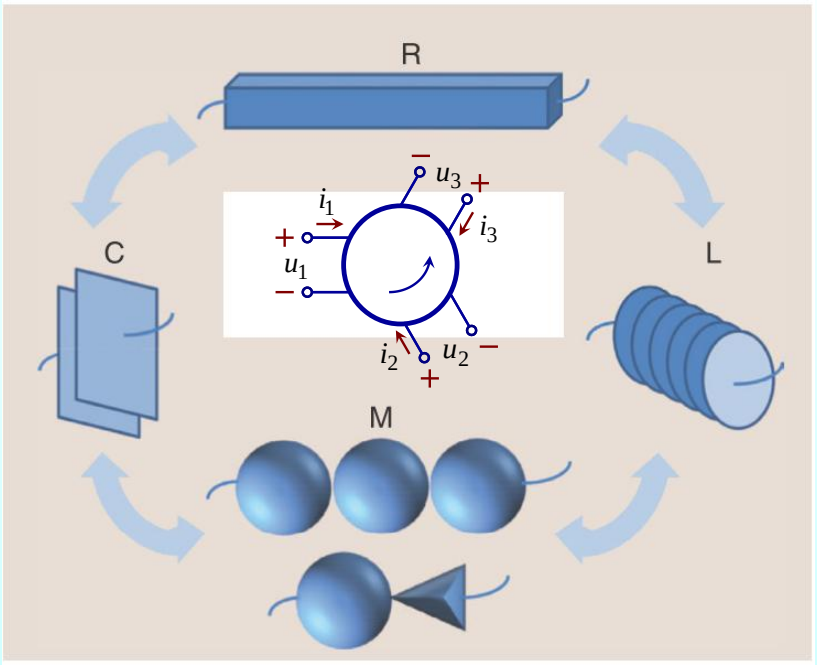
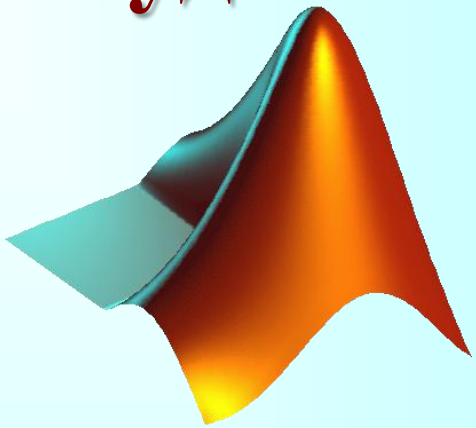
Практикум из рачунарске анализе кола

3. Временски домен. Побуде.





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Рачунарски (софтверски) алати

- *Mathematica*, MuPAD, **Maxima**, **SymPy**
- MATLAB, **Scilab**, **Octave**, **FreeMat**
- LTspice, **ngspice**, **QucsStudio**, **XCircuit**
- **Python**, MathCAD, MAPLE, SpeQ Mathematics…

Free/Libre Open Source Software (FLOSS)

Одзив на побуду

Вредности елемената електричног кола са слике су познате.

(а) Одредити једначине стања у скаларном и матричном облику. Који је ред кола?

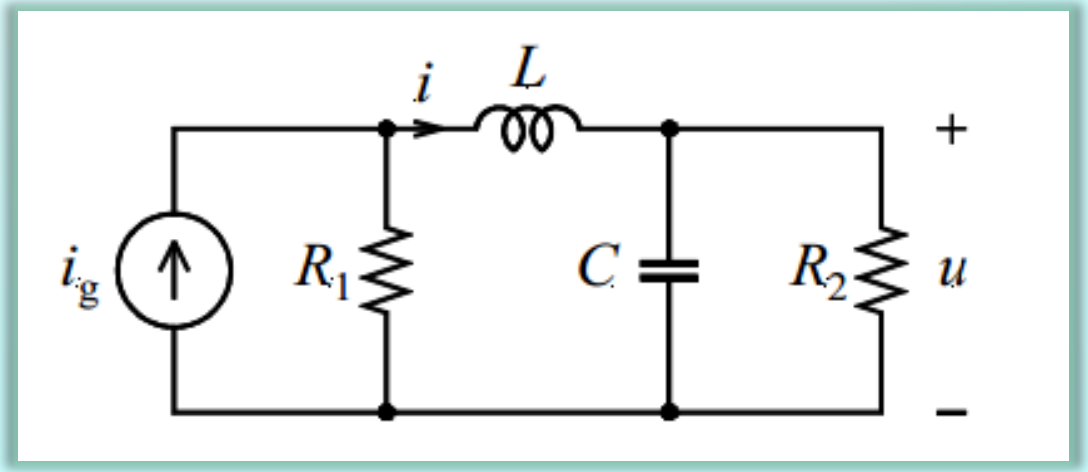
(б) Одредити импулсни одзив (Гринову функцију) за напон u ако је

$R_1 = R_2 = R$,

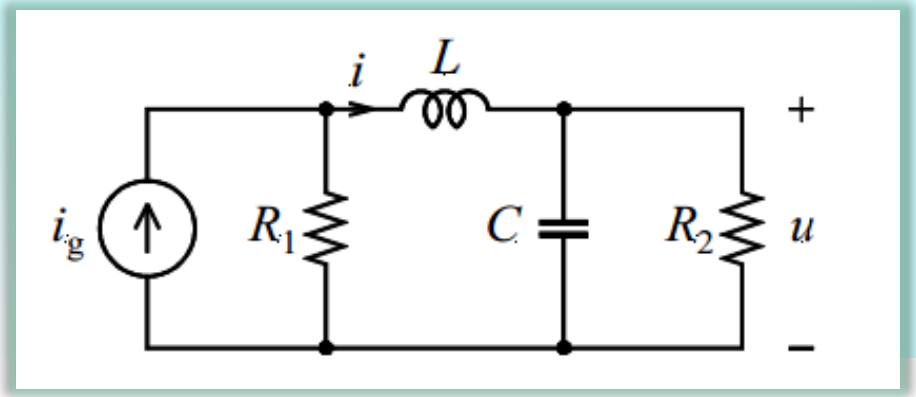
$L = CR^2$.

Нумерички пример:

$R_1 = R_2 = R = 1\ \text{k}\Omega$
$L = 1\ \text{H}$
$C = 1\ \mu\text{F}$
$i_g(t) = \delta(t)$



MATLAB: Symbolic Math Toolbox



Brisanje promejlivih na početku

```
clear variables
```

Definisanje simbola, odnosno nepoznatih

```
syms ig iL iR1 iR2 iC u1 u2 uL uC DiL DuC R1 R2 R C L t U
```

Jednacine

```
jednacine = [ig == iR1 + iL, iL == iC + iR2,...
    u1 == uL + uC, uC == u2,...
    u1 == R1*iR1, u2 == R2*iR2, uL == L*DiL, iC == C*DuC]
```

jednacine = $\left(\mathrm{ig} = \mathrm{iL} + \mathrm{iR}_1 \quad \mathrm{iL} = \mathrm{iC} + \mathrm{iR}_2 \quad u_1 = \mathrm{uC} + \mathrm{uL} \quad \mathrm{uC} = u_2 \quad u_1 = R_1\,\mathrm{iR}_1 \quad u_2 = R_2\,\mathrm{iR}_2 \quad \mathrm{uL} = \mathrm{DiL}\,L \quad \mathrm{iC} = C\,\mathrm{DuC}\right)$

```
jednacineIzvoda = eliminate(jednacine, [iR1, iR2, iC, u1, u2, uL])
```

jednacineIzvoda = $[\mathrm{uC} + R_1\,\mathrm{iL} - R_1\,\mathrm{ig} + \mathrm{DiL}\,L, R_2\,\mathrm{iL} - \mathrm{uC} - C\,\mathrm{DuC}\,R_2, R_1\,R_2\,\mathrm{ig} - R_2\,\mathrm{uC} - \mathrm{DiL}\,L\,R_2 - R_1\,\mathrm{uC} - C\,\mathrm{DuC}\,R_1\,R_2]$

Елиминација сувишних промењљивих и увођење функција

```
jednacineIzvoda = eliminate(jednacine, [iR1, iR2, iC, u1, u2, uL])
```

eliminate

$$\text{jednacineIzvoda} = \left[\text{uC} + R_1\,\text{iL} - R_1\,\text{ig} + \text{DiL}\,L,\ R_2\,\text{iL} - \text{uC} - C\,\text{DuC}\,R_2,\ R_1\,R_2\,\text{ig} - R_2\,\text{uC} - \text{DiL}\,L\,R_2 - R_1\,\text{uC} - C\,\text{DuC}\,R_1\,R_2\right]$$

```
izvodiStanja = solve(jednacineIzvoda, DuC, DiL)
```

```
izvodiStanja = struct with fields:
    DuC: -(uC - R2*iL)/(C*R2)
    DiL: -(uC + R1*iL - R1*ig)/L
```

```
syms iL(t) uC(t) DuC(t) DiL(t) ig(t) g_uC(t) f_uC(t)
promenljiveStanja = [iL == iL(t), DiL == diff(iL(t)), uC==uC(t), DuC==diff(uC(t))]
```

$$\text{promenljiveStanja}(t) = \left(\text{iL}(t) = \text{iL}(t)\quad \text{DiL}(t) = \frac{\partial}{\partial t}\,\text{iL}(t)\quad \text{uC}(t) = \text{uC}(t)\quad \text{DuC}(t) = \frac{\partial}{\partial t}\,\text{uC}(t)\right)$$

```
jednacineStanjaFun = subs([diff(uC)==izvodiStanja.DuC; ...
                           diff(iL)==izvodiStanja.DiL;],...
                           lhs(promenljiveStanja), rhs(promenljiveStanja))
```

$$\text{jednacineStanjaFun}(t) = \begin{pmatrix} \frac{\partial}{\partial t}\,\text{uC}(t) = -\frac{\text{uC}(t) - R_2\,\text{iL}(t)}{C\,R_2} \\ \frac{\partial}{\partial t}\,\text{iL}(t) = -\frac{\text{uC}(t) - R_1\,\text{ig} + R_1\,\text{iL}(t)}{L} \end{pmatrix}$$

MATLAB: Symbolic Math Toolbox

MATLAB: Symbolic Math Toolbox

Замене

Zamene

```
zamene = [R1 == R, R2 == R, L == C*R^2, ig == heaviside(t)]
```

zamene(t) =

$$\left(R_1 = R \quad R_2 = R \quad L = C\,R^2 \quad \mathrm{ig}(t) = \mathrm{heaviside}(t)\right)$$

```
jednacineStanjaFunR = subs(jednacineStanjaFun, lhs(zamene), rhs(zamene))
```

jednacineStanjaFunR(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t}\,\mathrm{uC}(t) = -\frac{\mathrm{uC}(t) - R\,\mathrm{iL}(t)}{C\,R} \\ \frac{\partial}{\partial t}\,\mathrm{iL}(t) = -\frac{\mathrm{uC}(t) - R\,\mathrm{heaviside}(t) + R\,\mathrm{iL}(t)}{C\,R^2} \end{pmatrix}$$

Решавање система диференцијалних једначина и цртање графика функције

Resavanje sistema diferencijalnih jednacina

MATLAB: Symbolic Math Toolbox

```
assume(R>0 & C>0 & L>0 );
resenjeDiff = dsolve(jednacineStanjaFunR, [uC(0)==0; iL(0)==0])
```

```
resenjeDiff = struct with fields:
    iL: exp(-t/(C*R))*sin(t/(C*R))*(sign(t)/4 - (exp(t/(C*R))*(sign(t)/2 + 1/2)*(cos(t/(C*R)) - sin(t/(C*R))))/2 + 1/
    uC: - R*exp(-t/(C*R))*sin(t/(C*R))*(sign(t)/4 - (exp(t/(C*R))*(cos(t/(C*R)) + sin(t/(C*R)))*(sign(t)/2 + 1/2))/2
```

```
f_uC = simplify(rewrite(resenjeDiff.uC, 'Heaviside'))
```

f_uC =

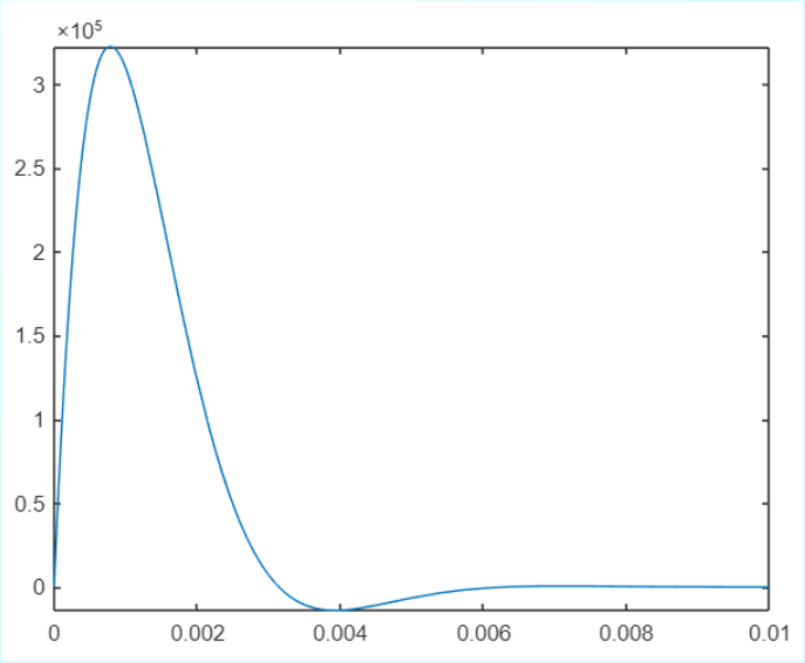
$$\frac{R\,\mathrm{heaviside}(t)}{2} - \frac{\sqrt{2}\,R\,e^{-\frac{t}{C\,R}}\sin\left(\frac{4\,t+\pi\,C\,R}{4\,C\,R}\right)\mathrm{heaviside}(t)}{2}$$

```
g_uC = simplify(diff(f_uC))
```

g_uC =

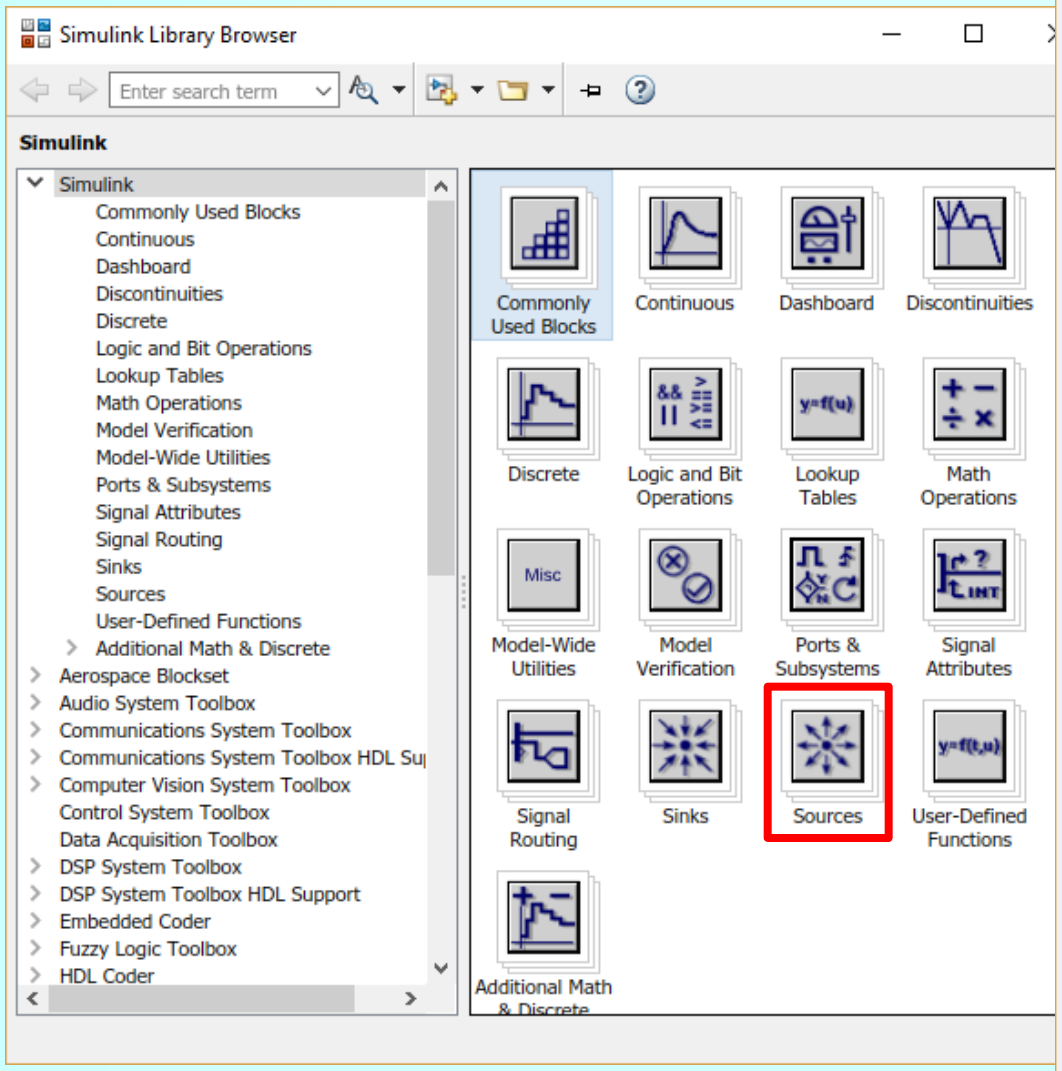
$$-\frac{e^{-\frac{t}{C\,R}}\cos\left(\frac{2\,t+\pi\,C\,R}{2\,C\,R}\right)\mathrm{heaviside}(t)}{C}$$

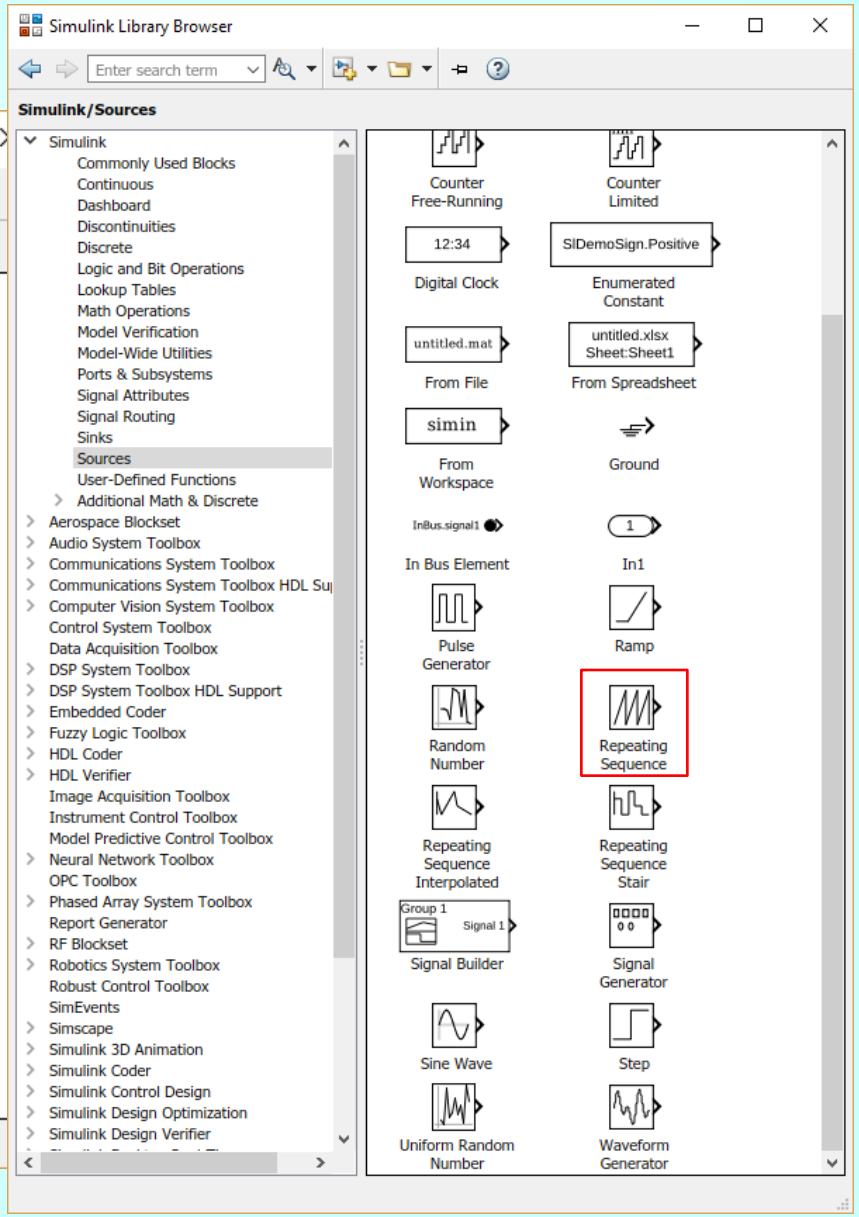
```
fplot(t, subs(g_uC, [R C], [1e3 1e-6]), [0 0.01])
```

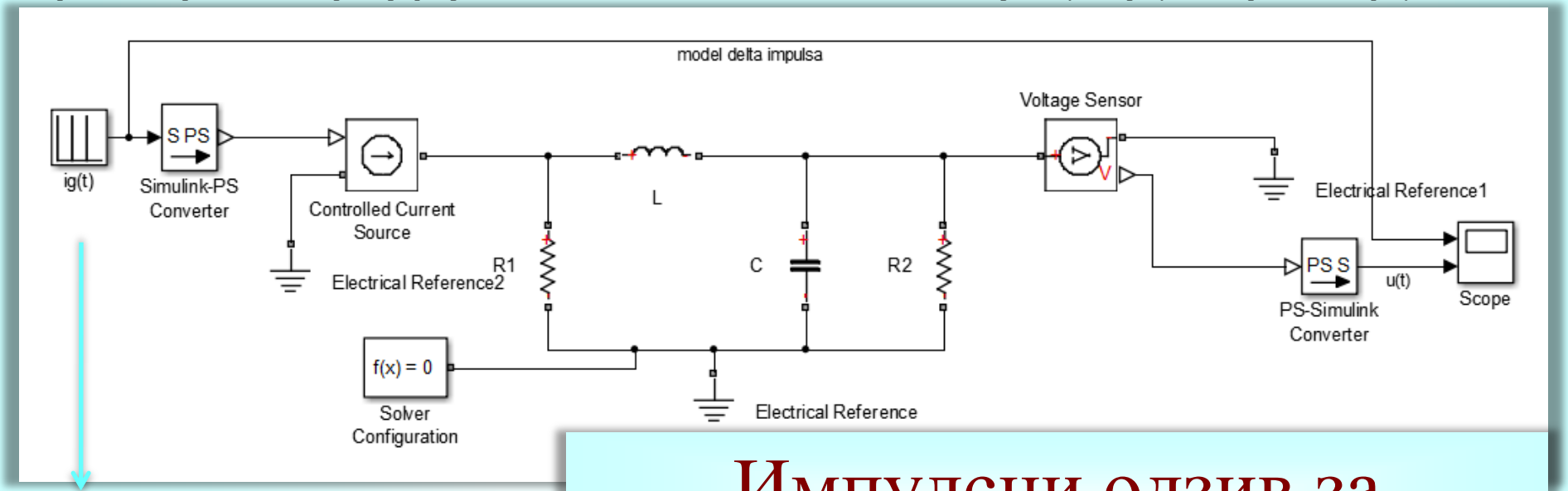


MATLAB: Simulink

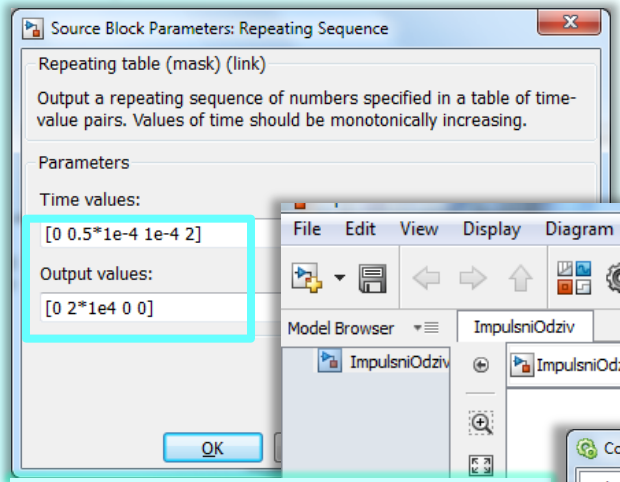
Sources

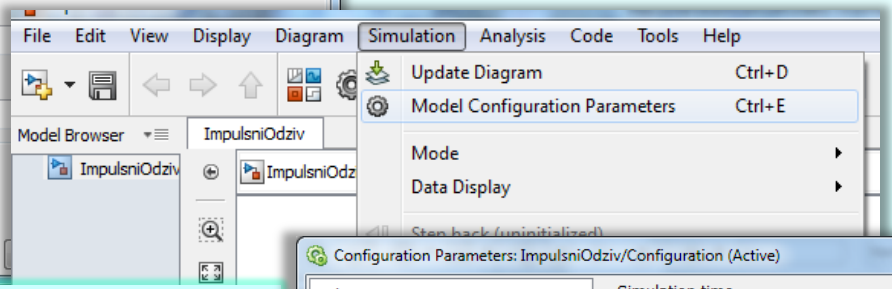




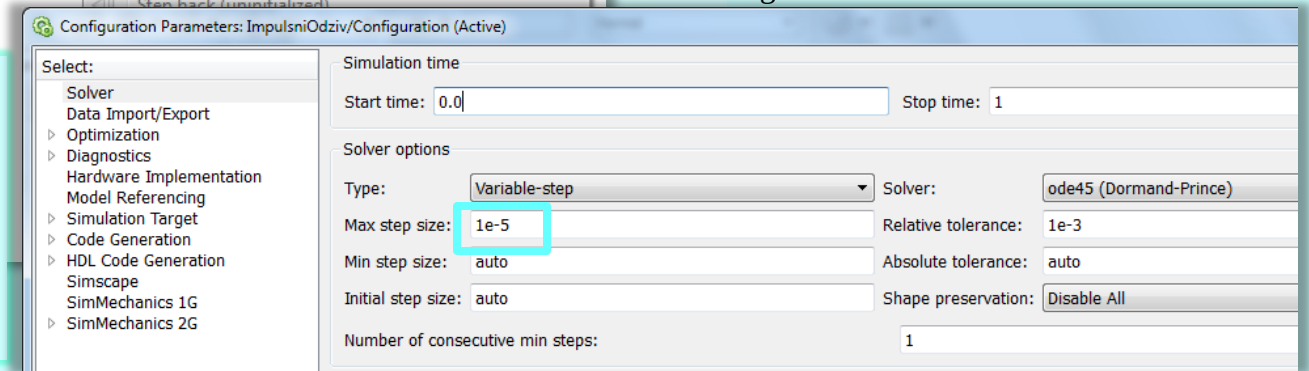


Импулсни одзив за напон кондензатора $u(t)$

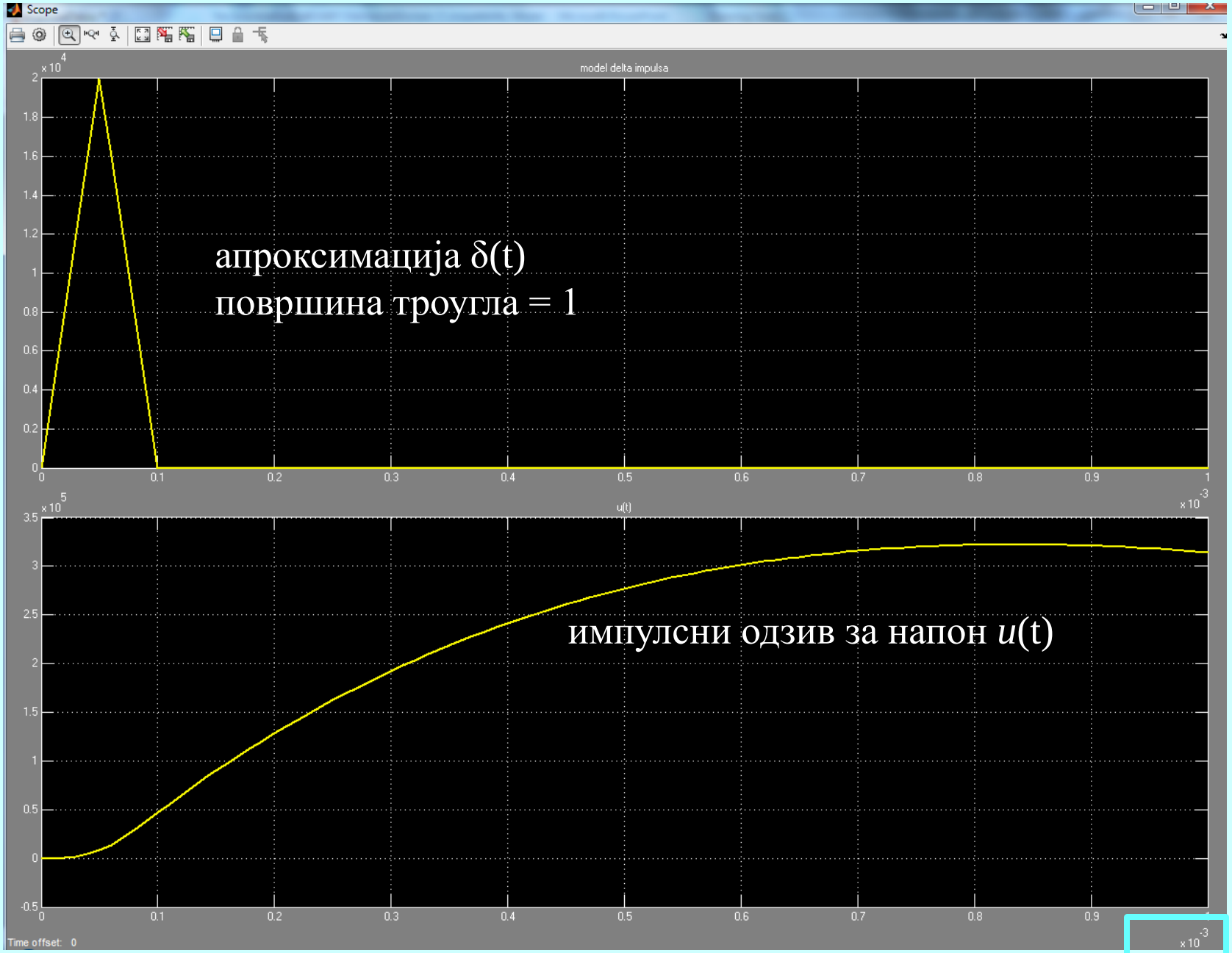


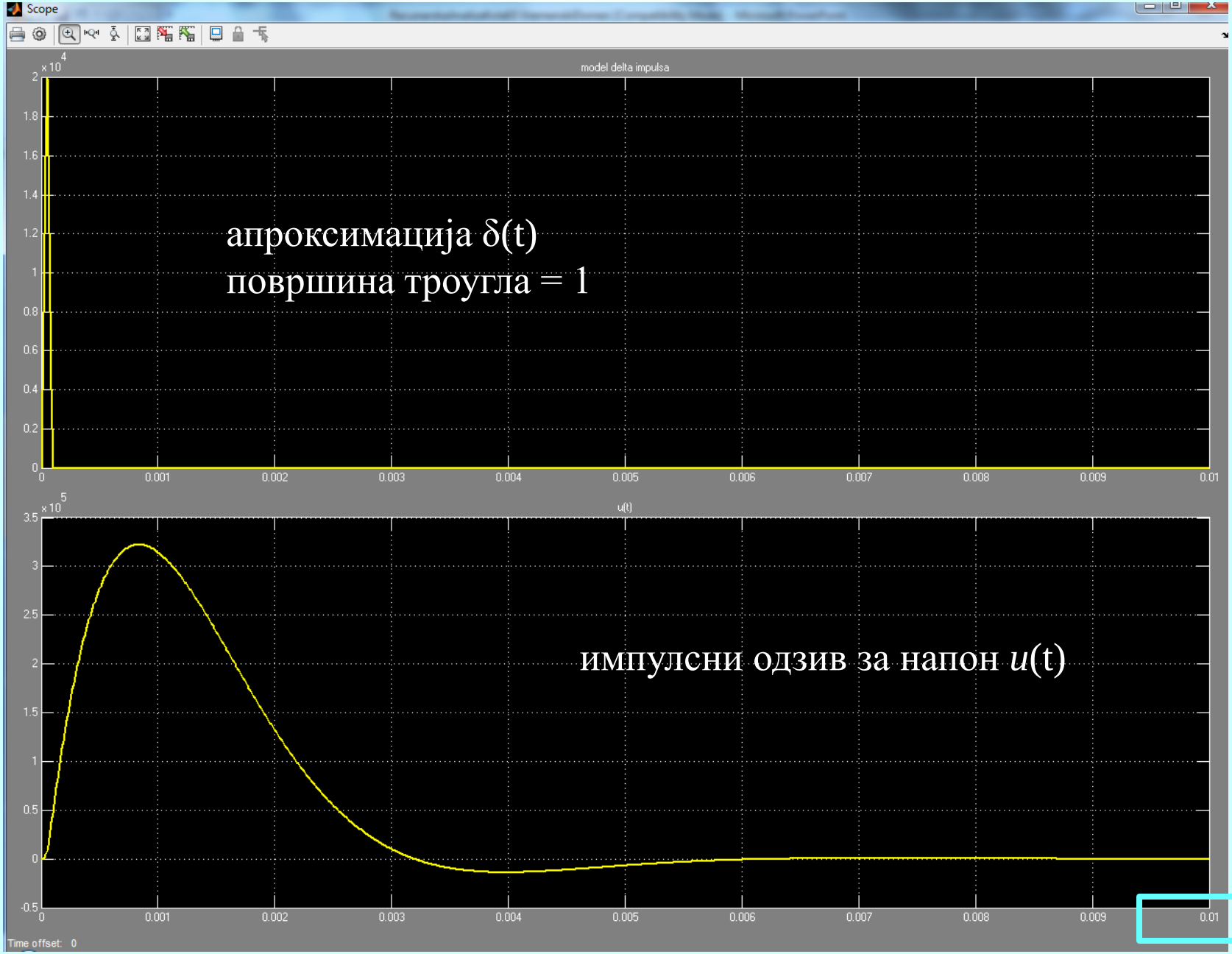


$R_1 = R_2 = R = 1\ \text{k}\Omega$
$L = 1\ \text{H}$
$C = 1\ \mu\text{F}$
$i_g(t) = \delta(t)$

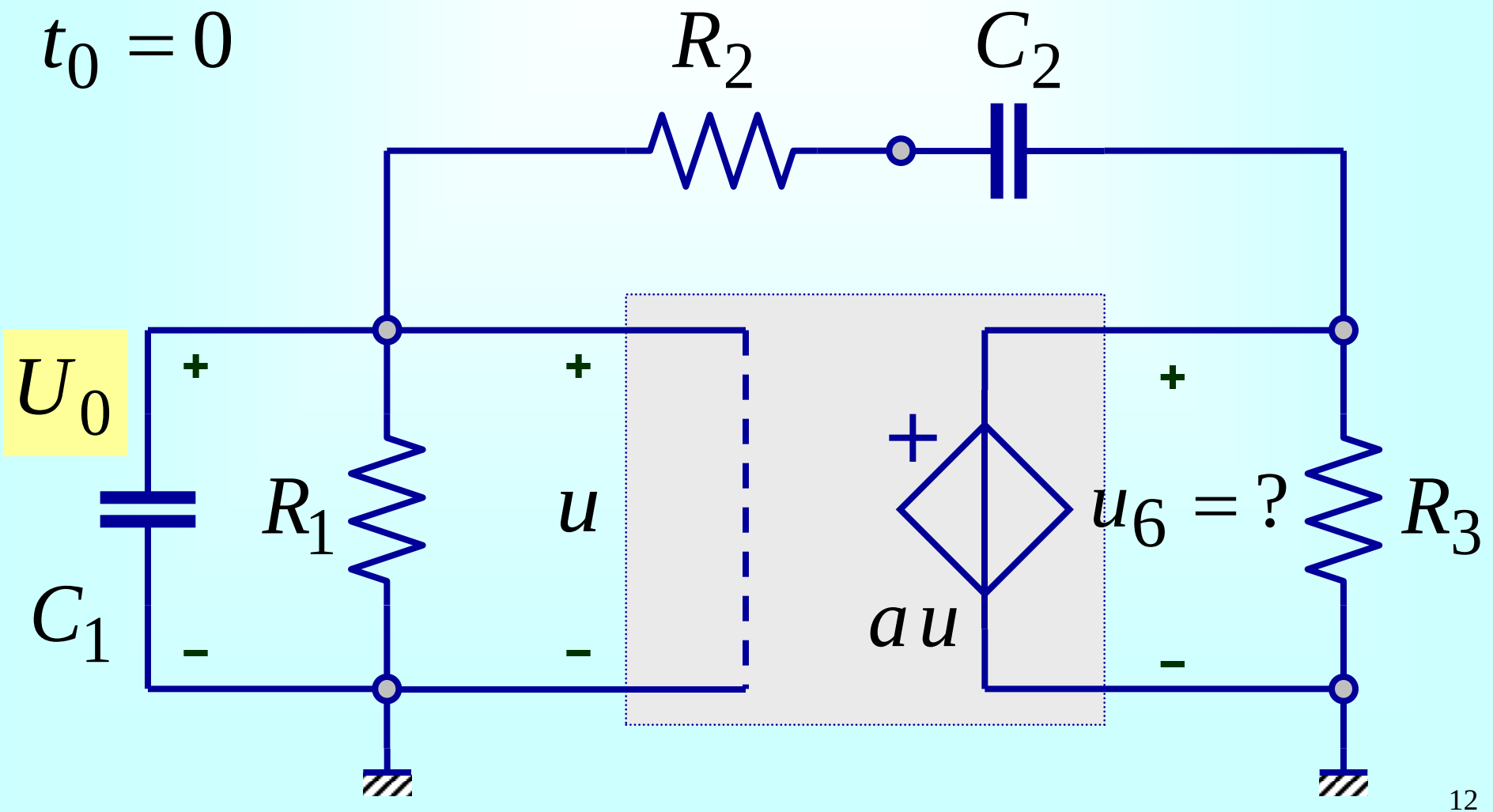


MATLAB: Simscape
Foundation Library
Utilities
Simulink

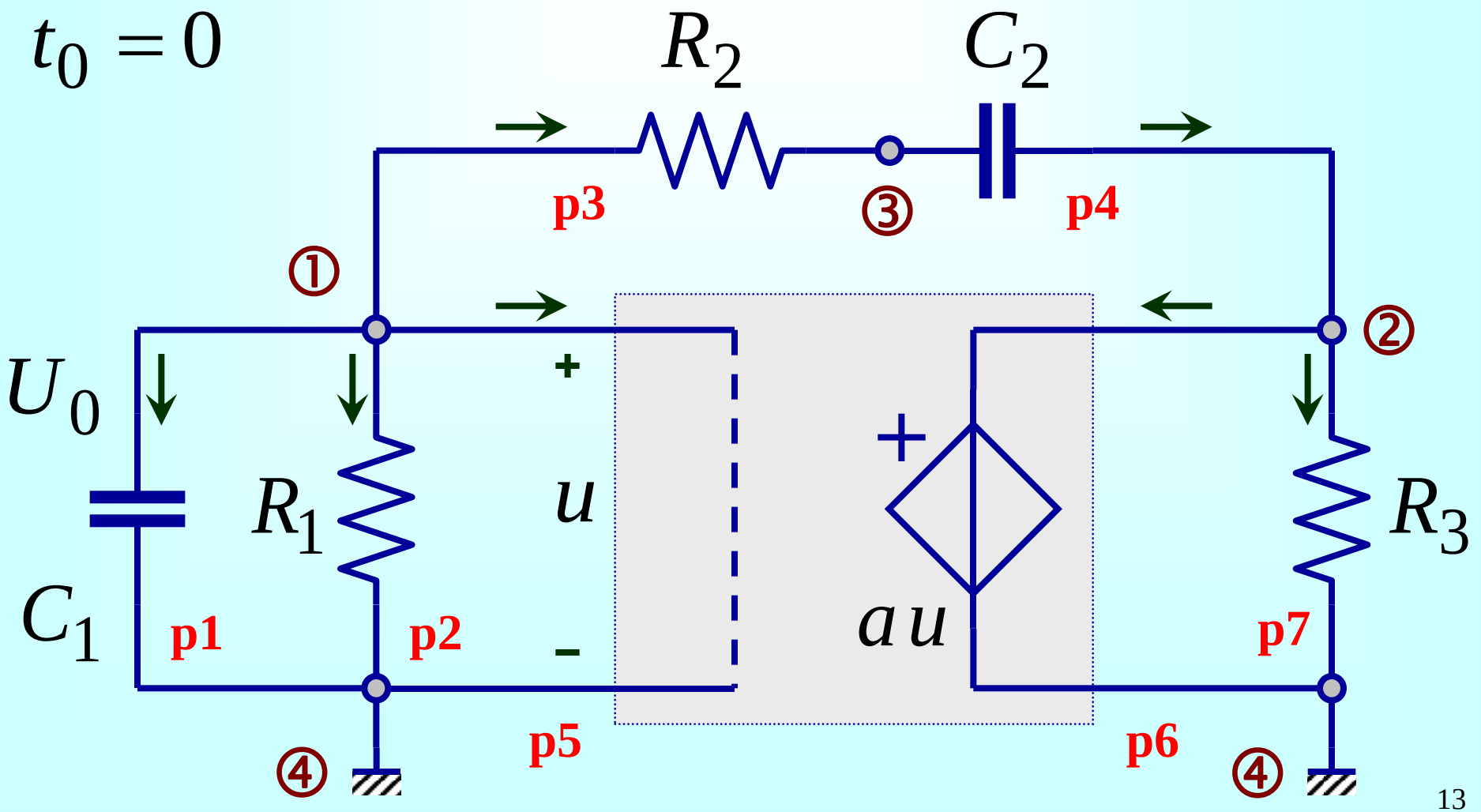

Scope
model delta impulsa
x 10^4
апроксимација δ(t)
површина троугла = 1
u(t)
x 10^5
x 10^-3
импулсни одзив за напон u(t)
Time offset: 0


Scope
model delta impulsa
x 10^4
апроксимација δ(t)
површина троугла = 1
u(t)
x 10^5
импулсни одзив за напон u(t)
Time offset: 0

Одзив на сакупљену енергију



Ознаке чворова и приступа



MATLAB: Symbolic Math Toolbox

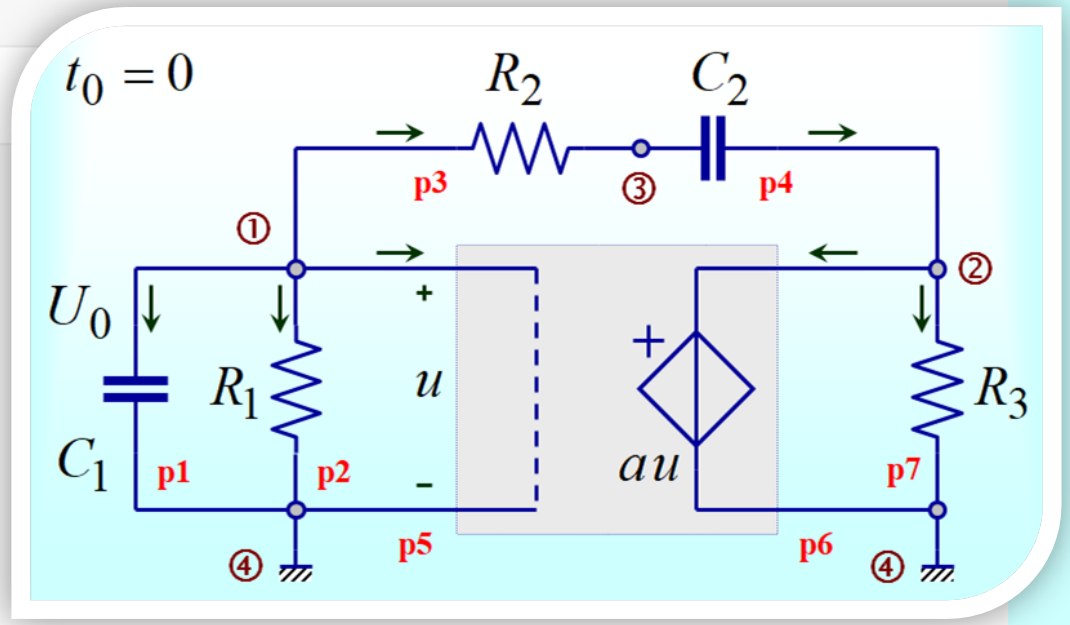
Систем једначина кола

Definisanje simbola, odnosno nepoznatih

```
syms i1 i2 i3 i4 i5 i6 i7 u1 u2 u3 u4 u5 u6 u7
syms Du1 Du4 D R1 R2 R3 R C1 C2 C a U0
```

Jednacine

```
jednacine  = [i1 + i2 + i3 + i5 == 0,...
    -i4 + i6 + i7 == 0,...
    -i3 + i4 == 0,...
    u1 - u2 == 0,...
    u2 - u5 == 0,...
    u5 - u6 - u4 - u3 == 0,...
    u6 - u7 == 0,...
    i1 == C1*Du1,...
    u2 == R1*i2,...
    u3 == R2*i3,...
    i4 == C2*Du4,...
    i5 == 0,...
    u6 == a*u5,...
    u7 == R3*i7]
```



$$\text{jednacine} = \left(i_1 + i_2 + i_3 + i_5 = 0 \quad i_6 - i_4 + i_7 = 0 \quad i_4 - i_3 = 0 \quad u_1 - u_2 = 0 \quad u_2 - u_5 = 0 \quad u_5 - u_4 - u_3 - u_6 = 0 \quad u_6 - u_7 =\right.$$

MATLAB: Symbolic Math Toolbox

Једначине стања

```
jednacineIzvoda = eliminate(jednacine, [i1, i2, i3, i4, i5, i6, i7, u2, u3, u5, u6, u7])
```

jednacineIzvoda = $\left[u_1 + C_1\,\mathrm{Du}_1\,R_1 + C_2\,\mathrm{Du}_4\,R_1,\; u_4 - u_1 + a\,u_1 + C_2\,\mathrm{Du}_4\,R_2\right]$

```
jednacineStanja = solve(jednacineIzvoda, [Du1, Du4])
```

```
jednacineStanja = struct with fields:
    Du1: -(R1*u1 + R2*u1 - R1*u4 - R1*a*u1)/(C1*R1*R2)
    Du4: -(u4 - u1 + a*u1)/(C2*R2)
```

Решавање система диференцијалних једначина

Resavanje sistema diferencijalnih jednacina

MATLAB: Symbolic Math Toolbox

```
syms u1(t) u4(t) Du1(t) Du4(t)
promenljiveStanja = [u1 == u1(t) u4 == u4(t)]
```

promenljiveStanja(t) = $\left(u_1(t) = u_1(t) \quad u_4(t) = u_4(t)\right)$

```
zamene = [R1 == R, R2 == R, R3 == R, C1 ==C, C2 == C, a == 3]
```

zamene = $\left(R_1 = R \quad R_2 = R \quad R_3 = R \quad C_1 = C \quad C_2 = C \quad a = 3\right)$

```
jednacineStanjaFun = subs([diff(u1) == jednacineStanja.Du1;...
                           diff(u4) == jednacineStanja.Du4],...
                            [lhs(promenljiveStanja) lhs(zamene)], [rhs(promenljiveStanja) rhs(zamene)])
```

jednacineStanjaFun(t) =

$$\left(\frac{\partial}{\partial t}\, u_1(t) = \frac{R\, u_1(t) + R\, u_4(t)}{C\, R^2} \qquad \frac{\partial}{\partial t}\, u_4(t) = -\frac{2\, u_1(t) + u_4(t)}{C\, R}\right)$$

```
resenjeDiff = dsolve(jednacineStanjaFun, [u1(0)==10, u4(0)==0])
```

почетне вредности

```
resenjeDiff = struct with fields:
    u4: - exp(-(t*1i)/(C*R))*10i + exp((t*1i)/(C*R))*10i
    u1: exp(-(t*1i)/(C*R))*(5 + 5i) + exp((t*1i)/(C*R))*(5 - 5i)
```

MATLAB: Symbolic Math Toolbox

Одзив – напон $u_6(t)$

```
u6(t) = rewrite(simplify(3*resenjeDiff.u1), 'cos')
```

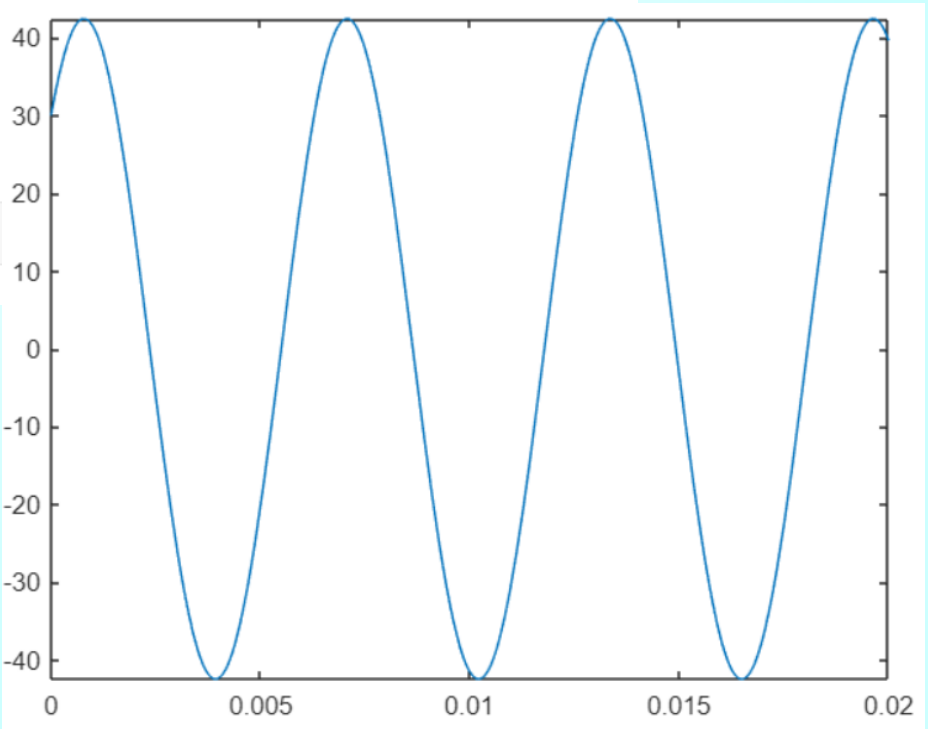
rewrite

u6(t) =

$$30\cos\left(\frac{t}{C\,R}\right)+30\sin\left(\frac{t}{C\,R}\right)$$

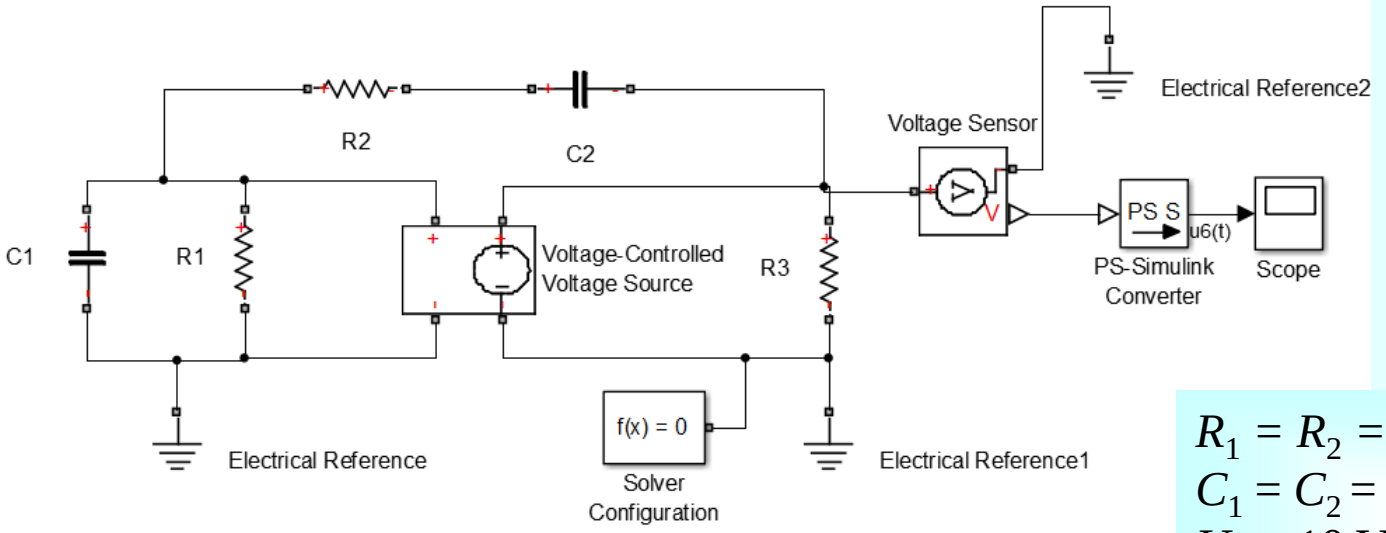
Crtanje dijagrama

```
fplot(t, subs(u6(t),[R, C], [1e3, 1e-6]), [0 20e-3])
```

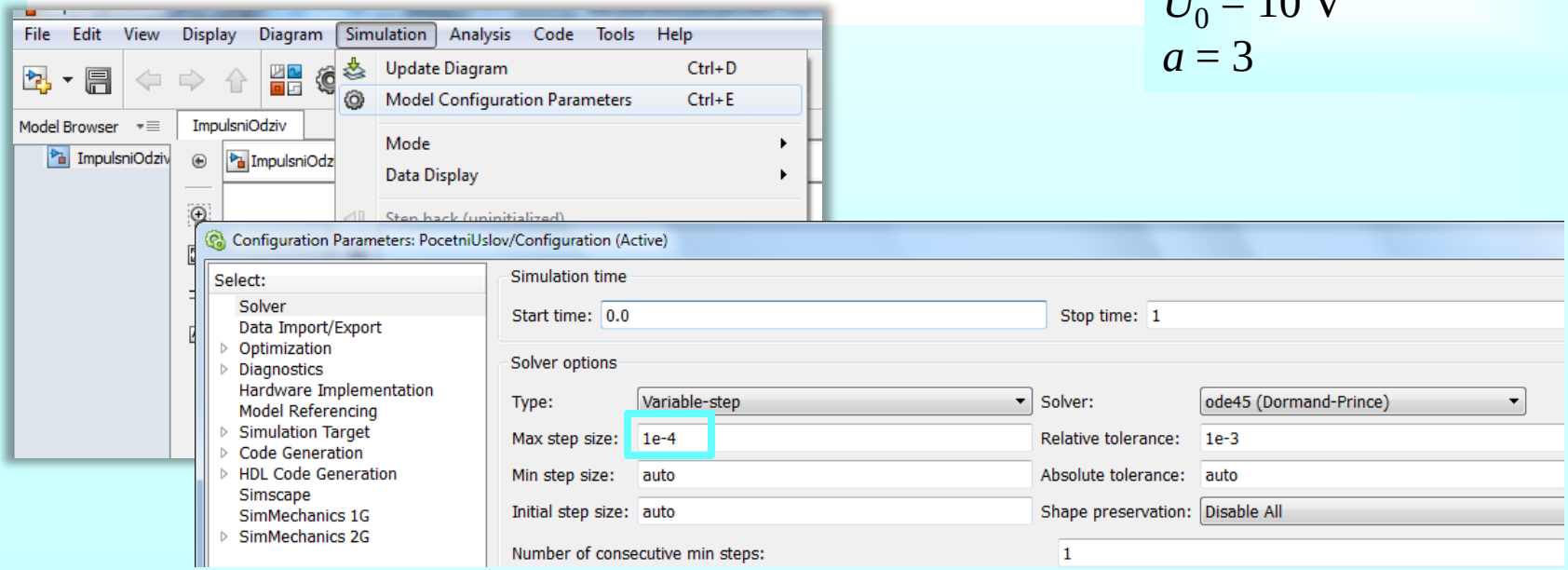


$t > t_0$

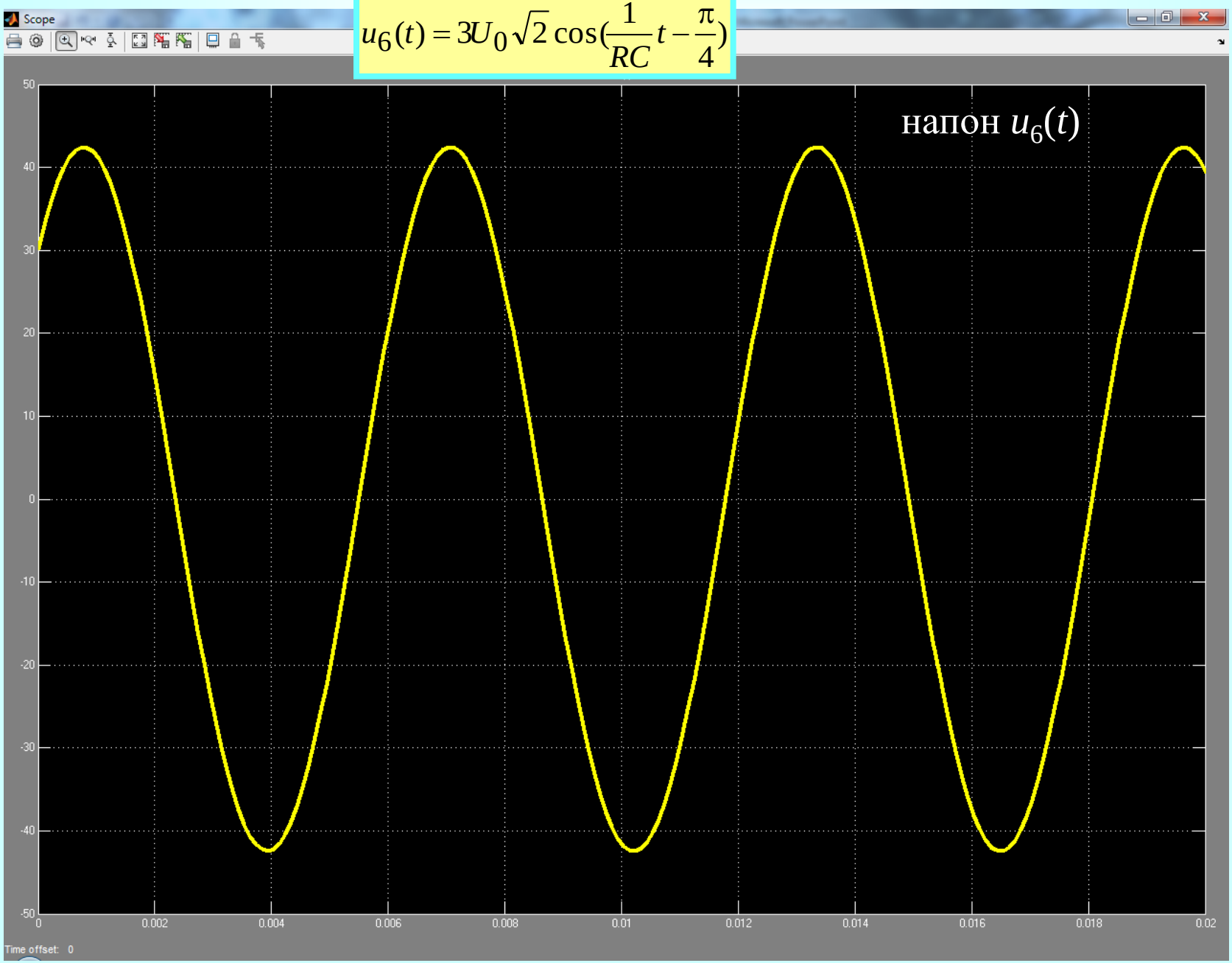
MATLAB: Simscape
Foundation Library, Utilities, Simulink



$R_1 = R_2 = R = 1\ \text{k}\Omega$
$C_1 = C_2 = 1\ \mu\text{F}$
$U_0 = 10\ \text{V}$
$a = 3$



$$u_6(t) = 3U_0\sqrt{2}\cos\left(\frac{1}{RC}t - \frac{\pi}{4}\right)$$



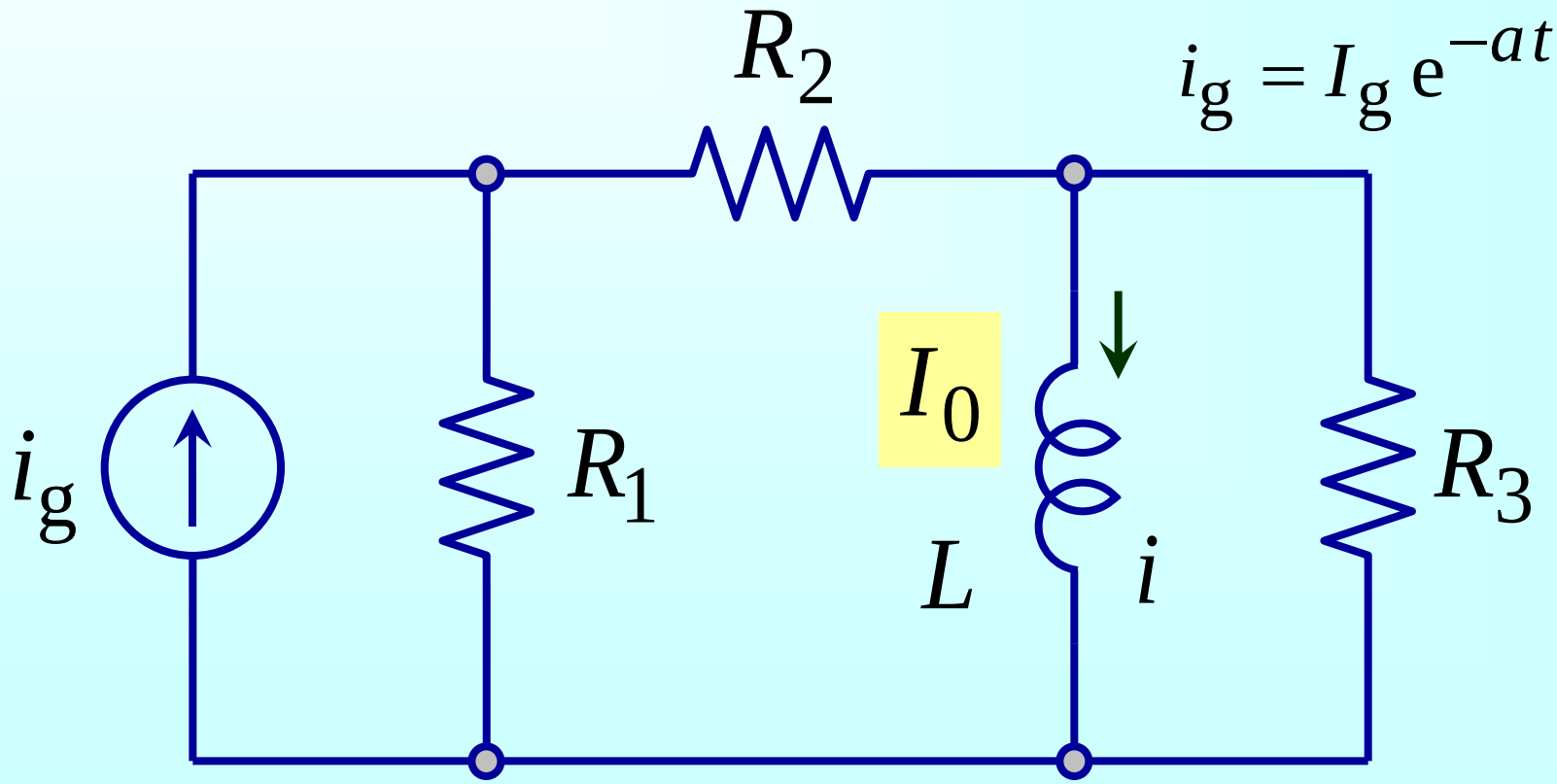
Одзив на побуду и сакупљену енергију

$t_0 = 0$

$R_1 = R_2 = R$

$R_3 = 2R$

$i_g = I_g \, e^{-at} \, h(t)$



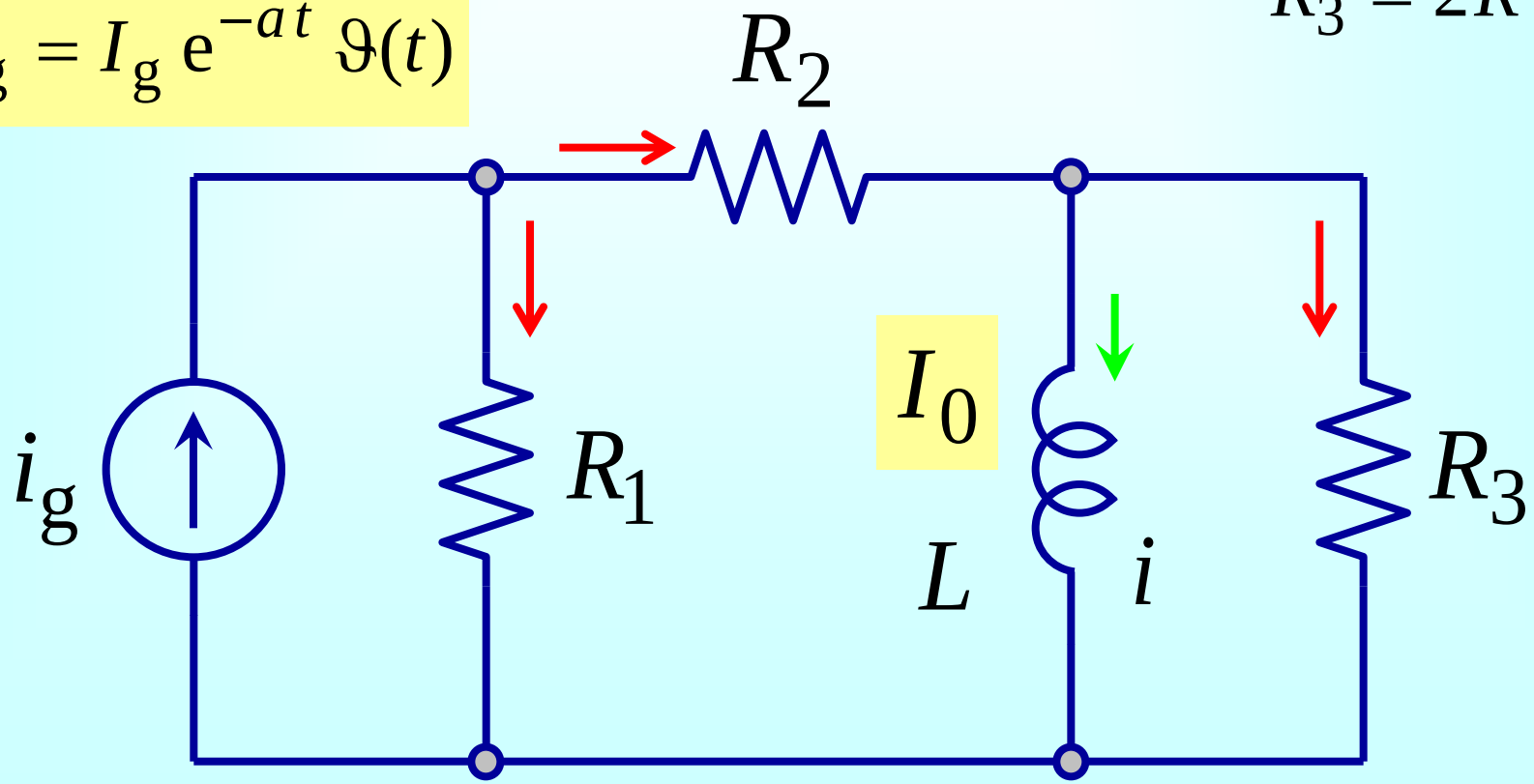
Коло са побудом и почет. енер.

$t_0 = 0$

$R_1 = R_2 = R$

$R_3 = 2R$

$i_g = I_g \, e^{-a\,t} \, \vartheta(t)$



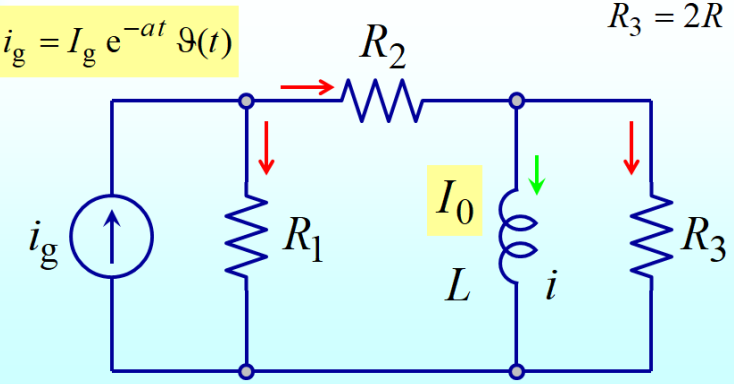
Систем једначина кола

$t_0 = 0$

$R_1 = R_2 = R$

$R_3 = 2R$

$i_g = I_g\, e^{-at}\, \vartheta(t)$



Definisanje simbola, odnosno nepoznatih

```
syms iR1 iR2 iR3 iL DiL uR1 uR2 uR3 uL ig Ig I0 a L R R1 R2 R3
```

Jednacine

```
jednacine  = [iR1 + iR2 - ig == 0,...
              -iR2 + iL + iR3 == 0,...
              uR1 - uL - uR2 == 0,...
              uL - uR3 == 0,...
              uL == L*DiL,...
              uR1 == R1*iR1,...
              uR2 == R2*iR2,...
              uR3 == R3*iR3]
```

jednacine = $\left(\mathrm{iR}_1 + \mathrm{iR}_2 - \mathrm{ig} = 0 \quad \mathrm{iL} - \mathrm{iR}_2 + \mathrm{iR}_3 = 0 \quad \mathrm{uR}_1 - \mathrm{uL} - \mathrm{uR}_2 = 0 \quad \mathrm{uL} - \mathrm{uR}_3 = 0 \quad \mathrm{uL} = \mathrm{DiL}\, L \quad \mathrm{uR}_1 = R_1\, \mathrm{iR}_1 \quad \mathrm{uR}_2 = R_2\, \mathrm{iR}_2 \quad \mathrm{uR}_3 = R_3\, \mathrm{iR}_3\right)$

```
jednacineIzvoda = eliminate(jednacine, [iR1, iR2, iR3, uR1, uR2, uR3, uL])
```

jednacineIzvoda = $\left[R_1 R_3\, \mathrm{ig} - \mathrm{DiL}\, L\, R_2 - \mathrm{DiL}\, L\, R_3 - R_1 R_3\, \mathrm{iL} - R_2 R_3\, \mathrm{iL} - \mathrm{DiL}\, L\, R_1\right]$

```
jednacineStanja = solve(jednacineIzvoda, DiL)
```

jednacineStanja =

$$-\frac{R_1 R_3\, \mathrm{iL} + R_2 R_3\, \mathrm{iL} - R_1 R_3\, \mathrm{ig}}{L R_1 + L R_2 + L R_3}$$

MATLAB: Symbolic Math Toolbox

Једначина стања

```
syms iL(t)
promenljiveStanja = [iL == iL(t)]
```

promenljiveStanja(t) = $\mathrm{iL}(t) = \mathrm{iL}(t)$

```
zamene = [R1 == R, R2 == R, R3 == 2*R, ig == Ig*exp(-a*t)*heaviside(t)]
```

zamene = $\left(R_1 = R \quad R_2 = R \quad R_3 = 2\,R \quad \mathrm{ig} = \mathrm{Ig}\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)\right)$

```
jednacineStanjaFun = subs(diff(iL(t))==jednacineStanja, lhs([promenljiveStanja zamene]), rhs([promenljiveStanja zamene]))
```

jednacineStanjaFun =

$$\frac{\partial}{\partial t}\,\mathrm{iL}(t) = -\frac{4\,R^2\,\mathrm{iL}(t) - 2\,\mathrm{Ig}\,R^2\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)}{4\,L\,R}$$

омогућава ефикасан испис решења помоћу Хевисајдових функција

```
resenjeDiff = dsolve(jednacineStanjaFun, iL(0)==I0, 'IgnoreAnalyticConstraints', false)
```

resenjeDiff =

$$I_0\,\mathrm{e}^{-\frac{R\,t}{L}} - \mathrm{e}^{-\frac{R\,t}{L}}\left(\frac{\mathrm{Ig}\,R\,\mathrm{heaviside}(t)}{2\,(R - L\,a)} - \frac{\mathrm{Ig}\,R\,\mathrm{e}^{\frac{R\,t}{L}}\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t)}{2\,(R - L\,a)}\right)$$

MATLAB: Symbolic Math Toolbox

Одзив – струја $i(t)$

```
numzamene = [I0 ==-10, R==1e3, L==1e-3, Ig==10 a == 100]
```

numzamene =

$$\left(I_0 = -10 \quad R = 1000 \quad L = \frac{1}{1000} \quad \mathrm{Ig} = 10 \quad a = 100\right)$$

```
f_iL(t) = subs(resenjeDiff, lhs(numzamene), rhs(numzamene))
```

f_iL(t) =

$$-10\,\mathrm{e}^{-1000000\,t} - \mathrm{e}^{-1000000\,t}\left(\frac{50000\,\mathrm{heaviside}(t)}{9999} - \frac{50000\,\mathrm{e}^{999900\,t}\,\mathrm{heaviside}(t)}{9999}\right)$$

велики број

```
f_iL_exp(t) = expand(f_iL(t))
```

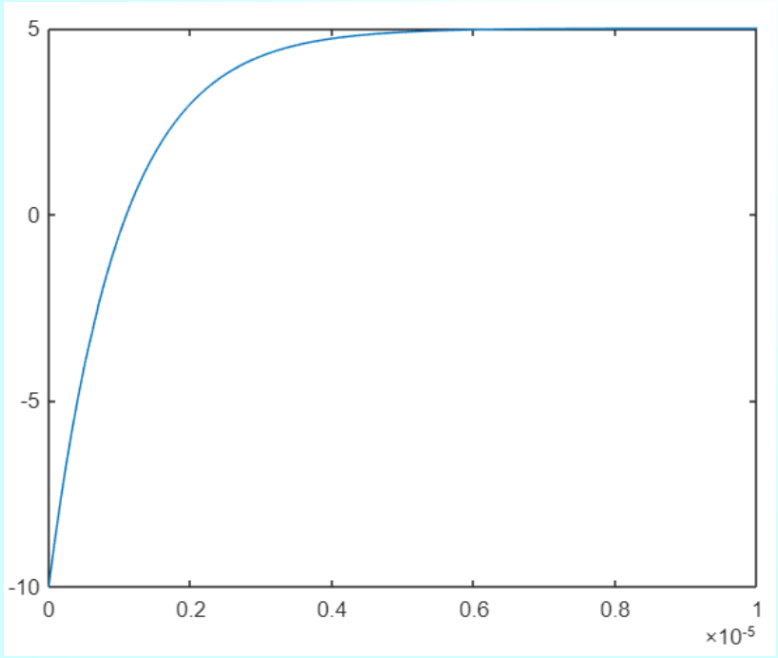
expand

f_iL_exp(t) =

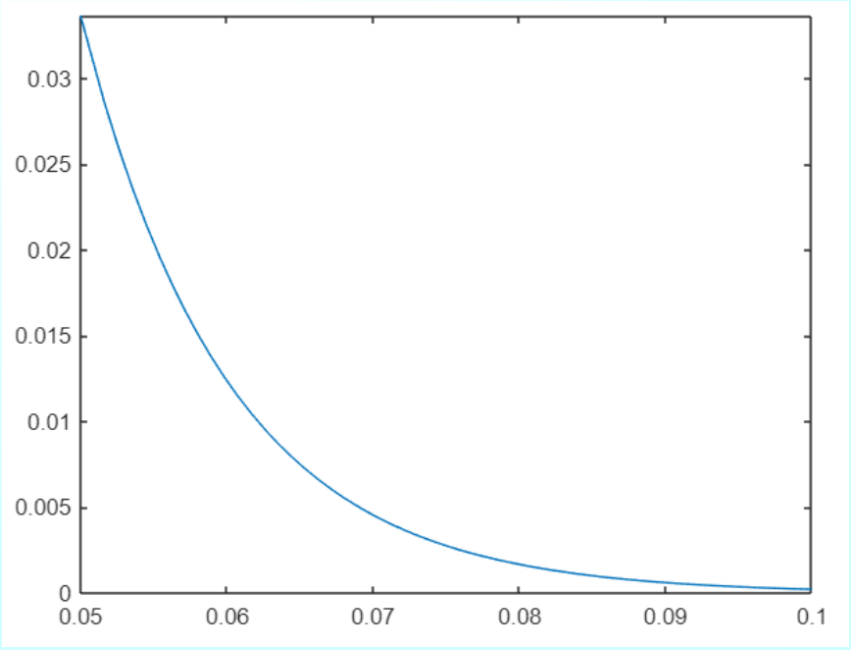
$$\frac{50000\,\mathrm{e}^{-100\,t}\,\mathrm{heaviside}(t)}{9999} - 10\,\mathrm{e}^{-1000000\,t} - \frac{50000\,\mathrm{e}^{-1000000\,t}\,\mathrm{heaviside}(t)}{9999}$$

MATLAB: Symbolic Math Toolbox

```
fplot(t, f_iL_exp(t), [0 10e-6])
```



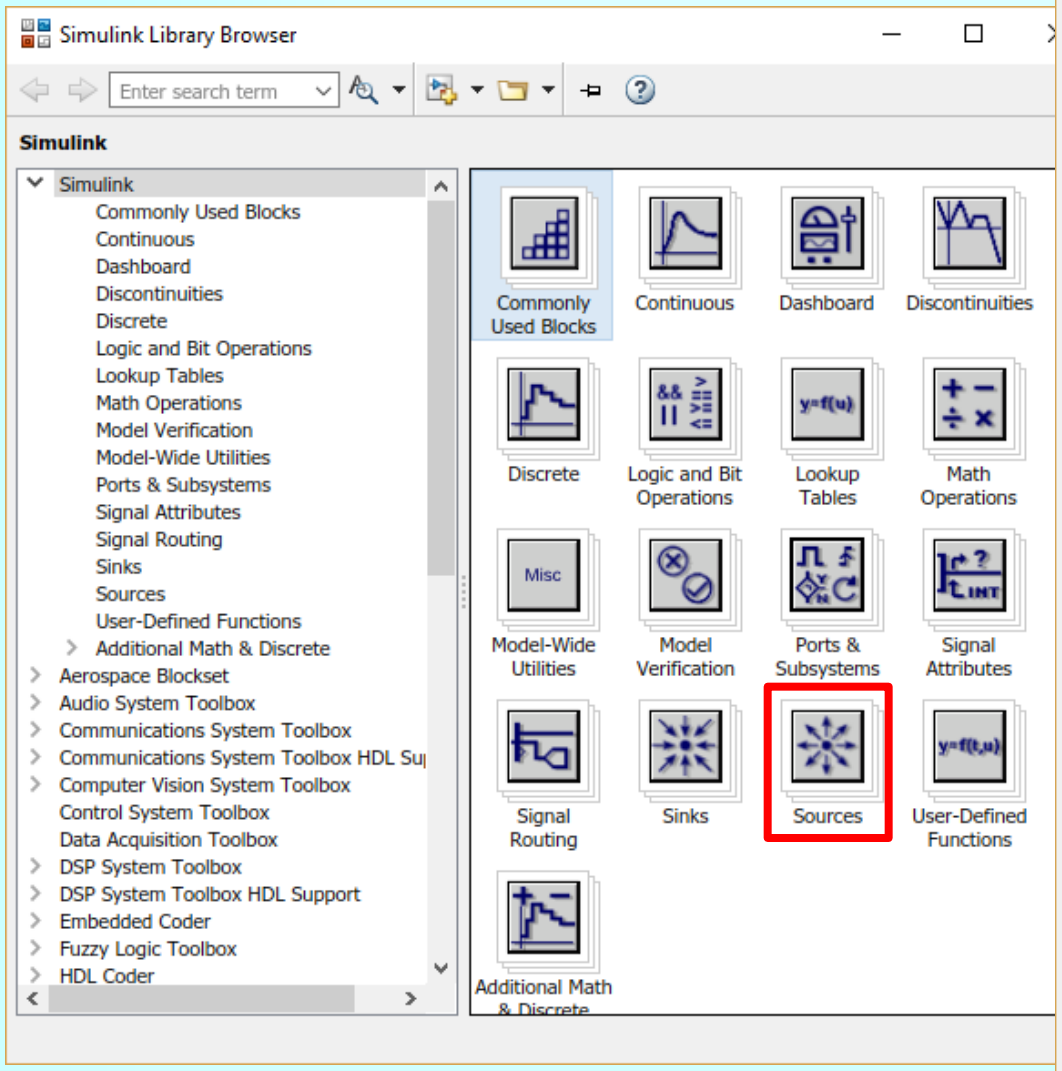

```
fplot(t, f_iL_exp(t), [0.05 0.1])
```

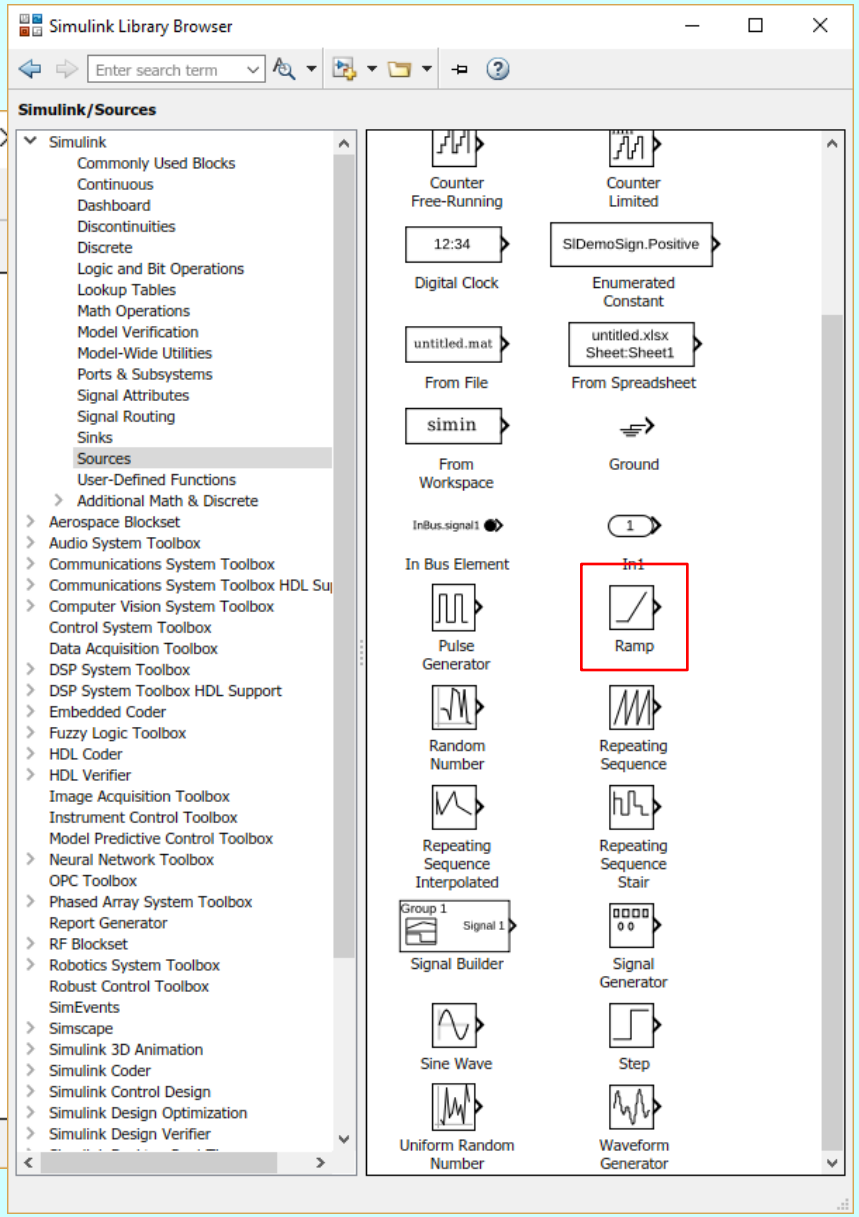


MATLAB: Symbolic Math Toolbox

MATLAB: Simulink

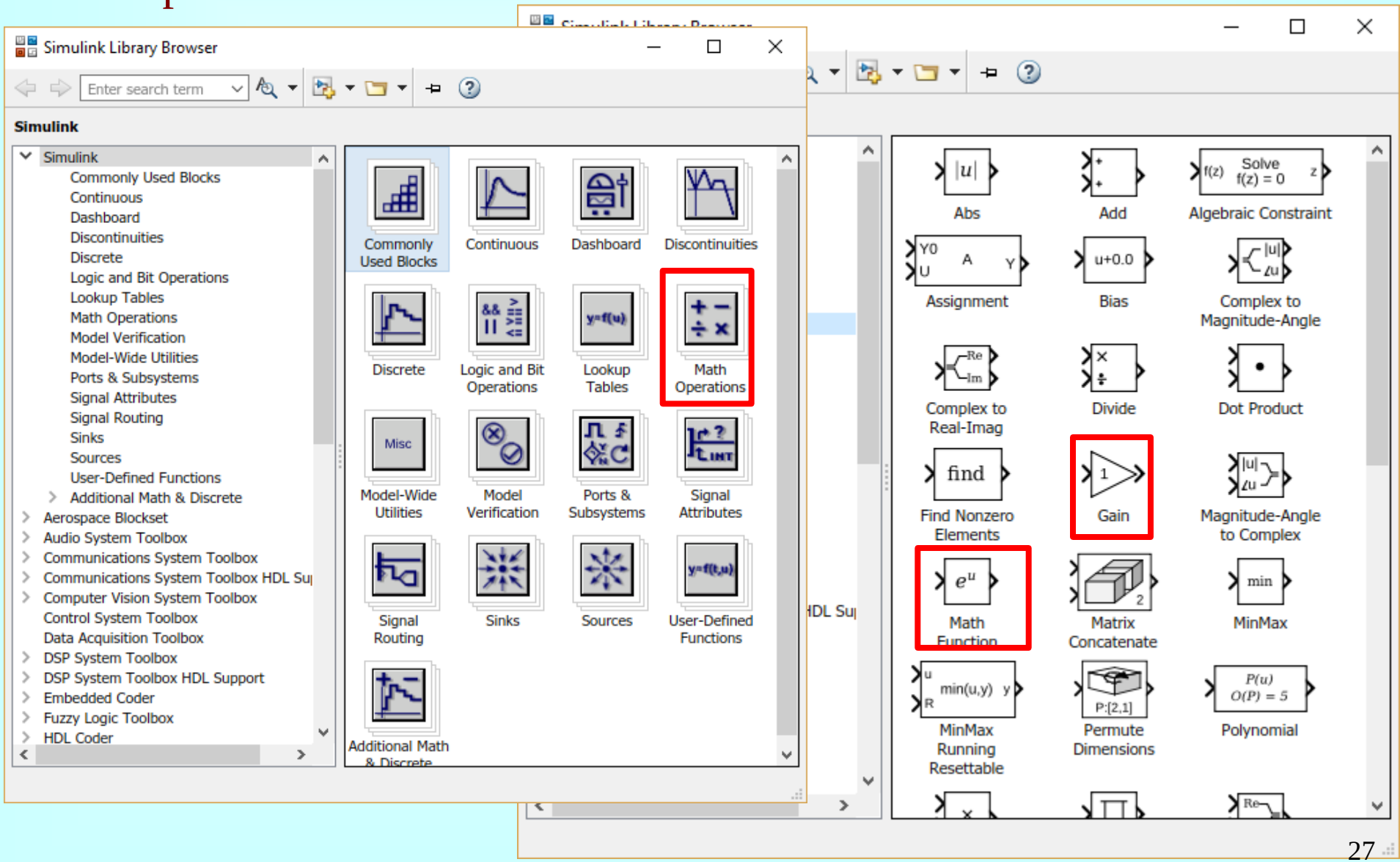
Sources



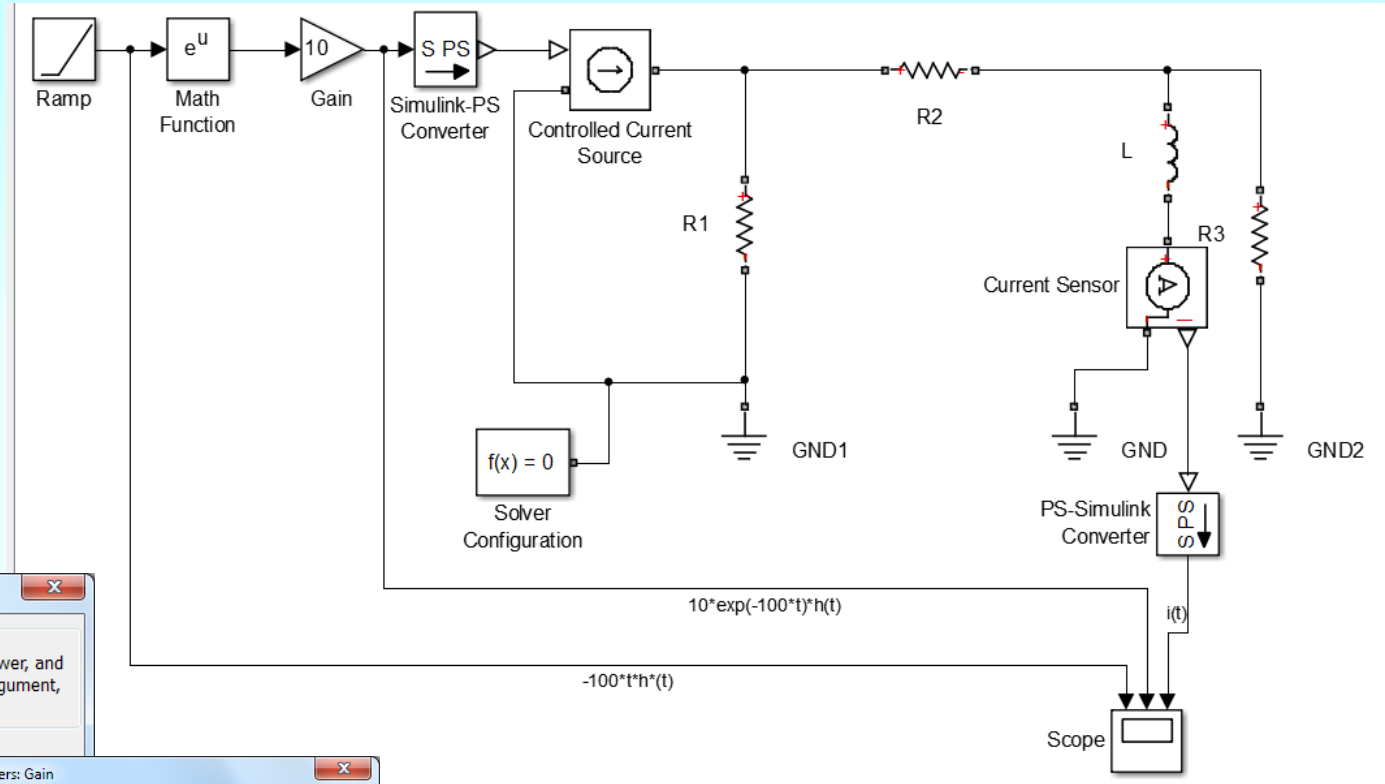


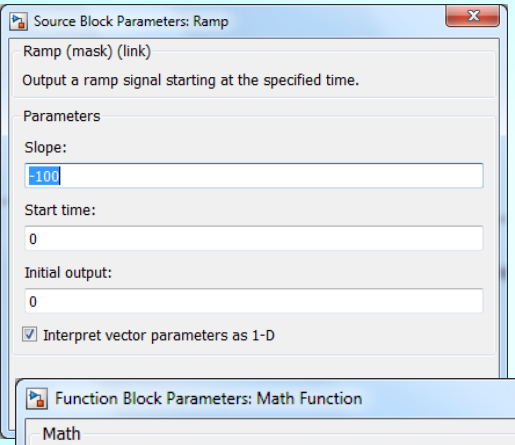
MATLAB: Simulink

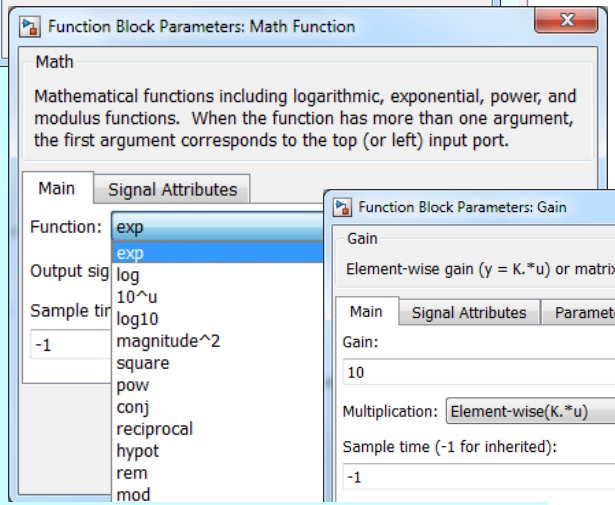
Math Operations

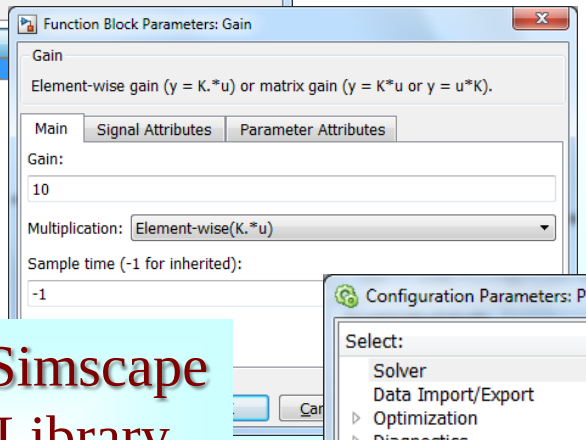


$$t > t_0, \quad t_0 = 0$$





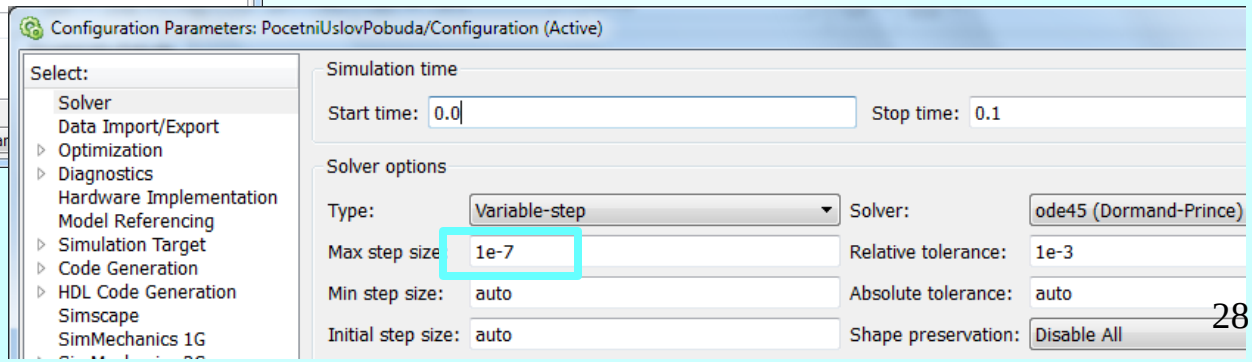




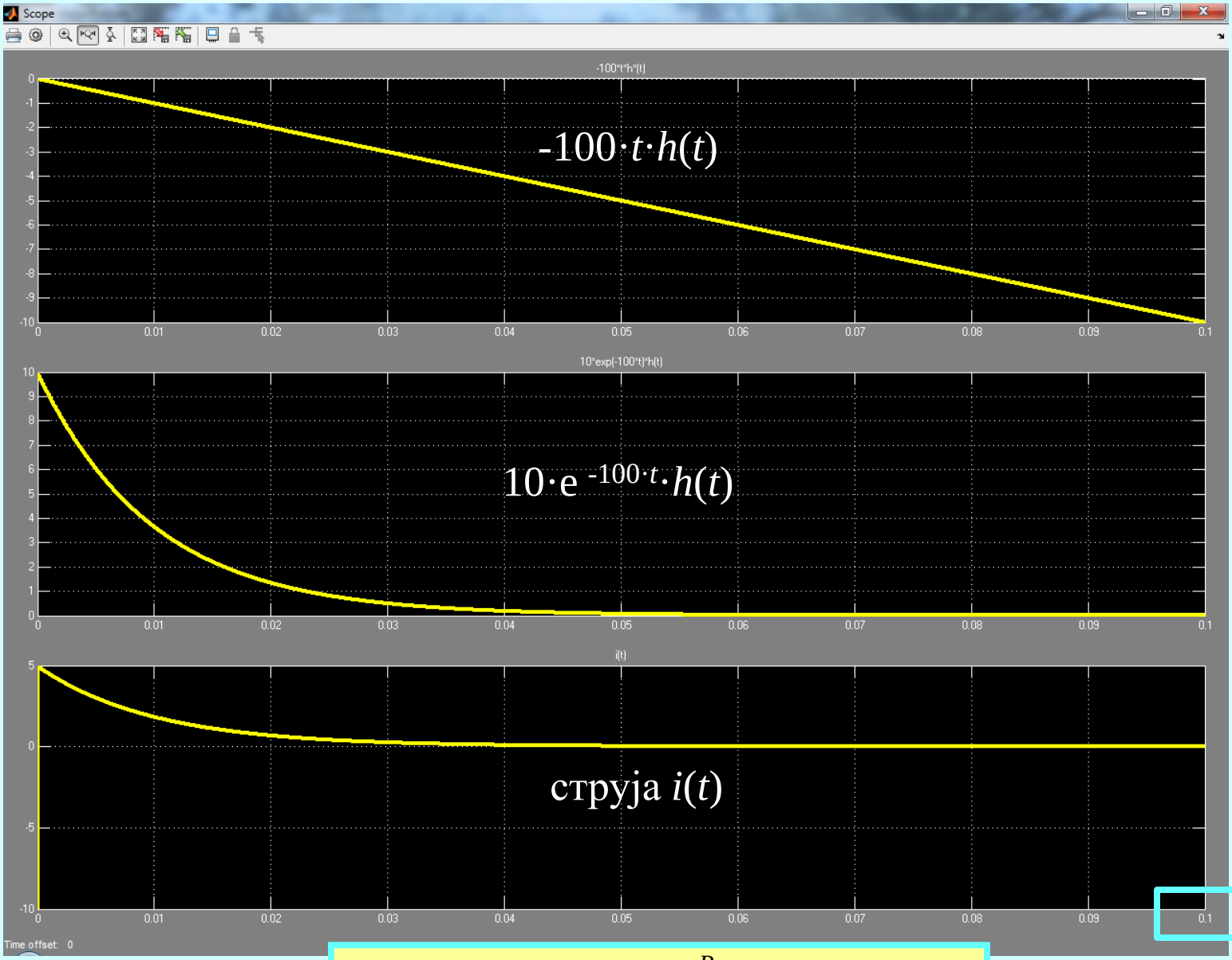
$$R_1 = R_2 = R = 1\text{k}\Omega, R_3 = 2R, L = 1\text{mH}$$

$$i_g(t) = I_g e^{-at} h(t), I_g = 10\text{ A}, a = 100$$

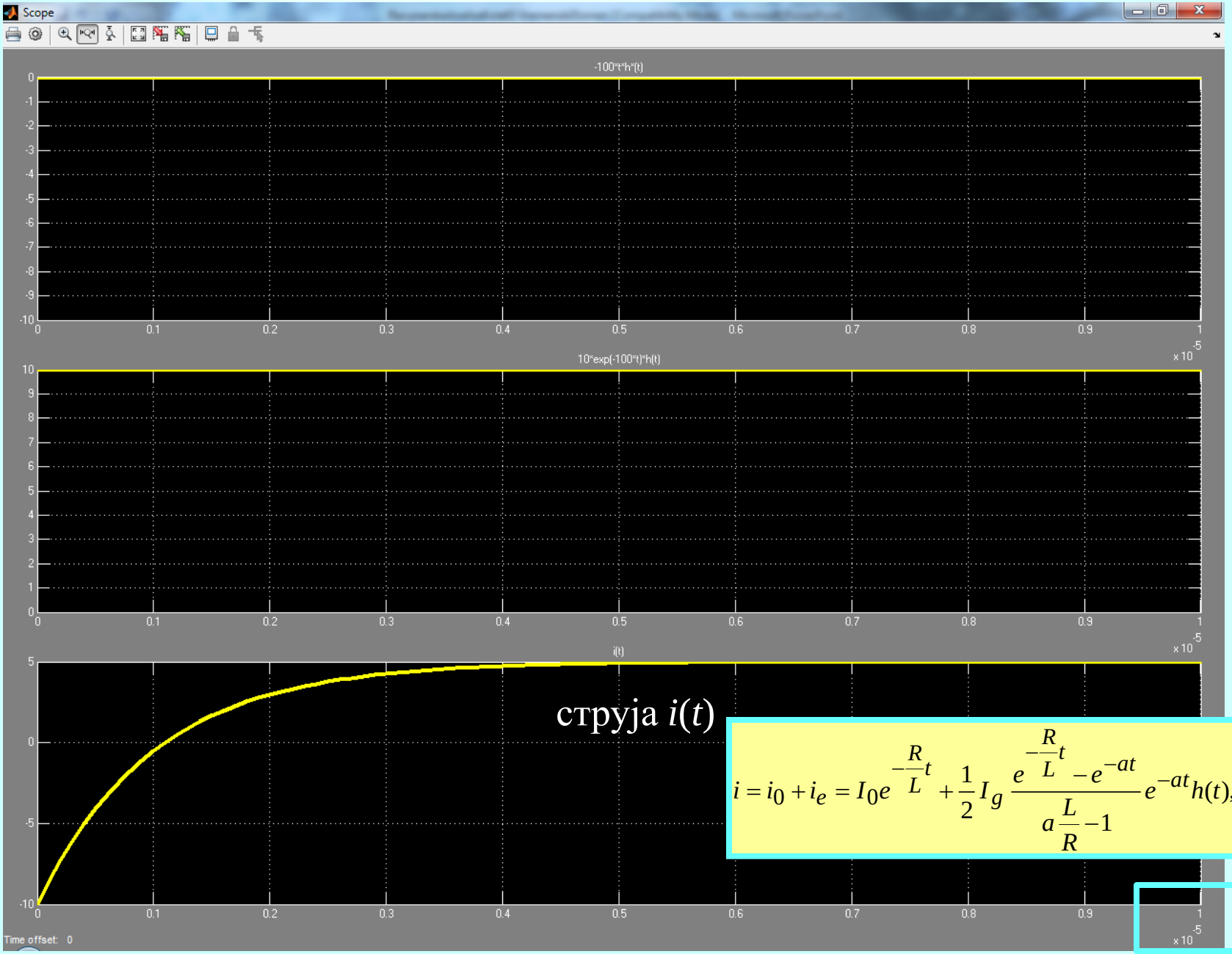
$$i(0^-) = I_0 = -10\text{ A}$$



MATLAB: Simscape
Foundation Library
Utilities
Simulink



$$i(t) = i_0 + i_e = I_0 e^{-\frac{R}{L}t} + \frac{1}{2} I_g \frac{e^{-\frac{R}{L}t} - e^{-at}}{a\frac{L}{R} - 1} e^{-at} h(t), a \neq \frac{R}{L}$$



струја $i(t)$

$$i = i_0 + i_e = I_0 e^{-\frac{R}{L}t} + \frac{1}{2} I_g \frac{e^{-\frac{R}{L}t} - e^{-at}}{a\frac{L}{R} - 1} e^{-at} h(t), a \neq \frac{R}{L}$$