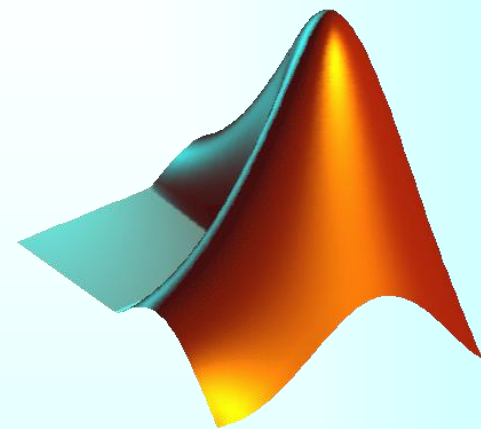
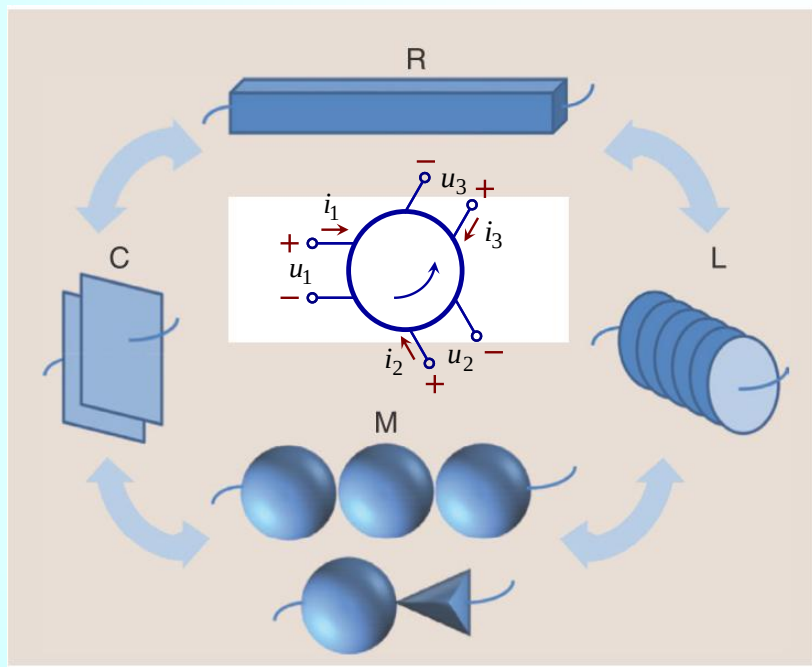


Рачунарска анализа електричних кола



Милка Потребих



Рачунарска анализа електричних кола на Енергетском одсеку

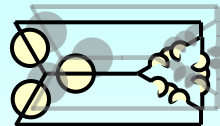
$$\underline{U}_0 = \frac{\underline{U}_a + \underline{U}_b + \underline{U}_c}{3}$$

$$\underline{U}_1 = \frac{\underline{U}_a + \underline{a}\underline{U}_b + \underline{a}^2\underline{U}_c}{3}$$

$$\underline{U}_2 = \frac{\underline{U}_a + \underline{a}^2\underline{U}_b + \underline{a}\underline{U}_c}{3}$$



Милка Потребих



Симетричне компоненте

временски непроменљивих, линеарних,
трофазних кола са устаљеним
простопериодичним одзивом

Задатак (1)

У трофазном електричном колу је дошло до квара и измерен је трофазни систем струја

$$\underline{I}_a = 150 \text{ A} \angle 45^\circ, \underline{I}_b = 250 \text{ A} \angle 150^\circ, \underline{I}_c = 100 \text{ A} \angle 300^\circ.$$

- (а) Одредити симетричне компоненте трофазног система струја.
- (б) Нацртати фазорски дијаграм симетричних компоненти.
- (в) Одредити тренутне вредности симетричних компоненти.

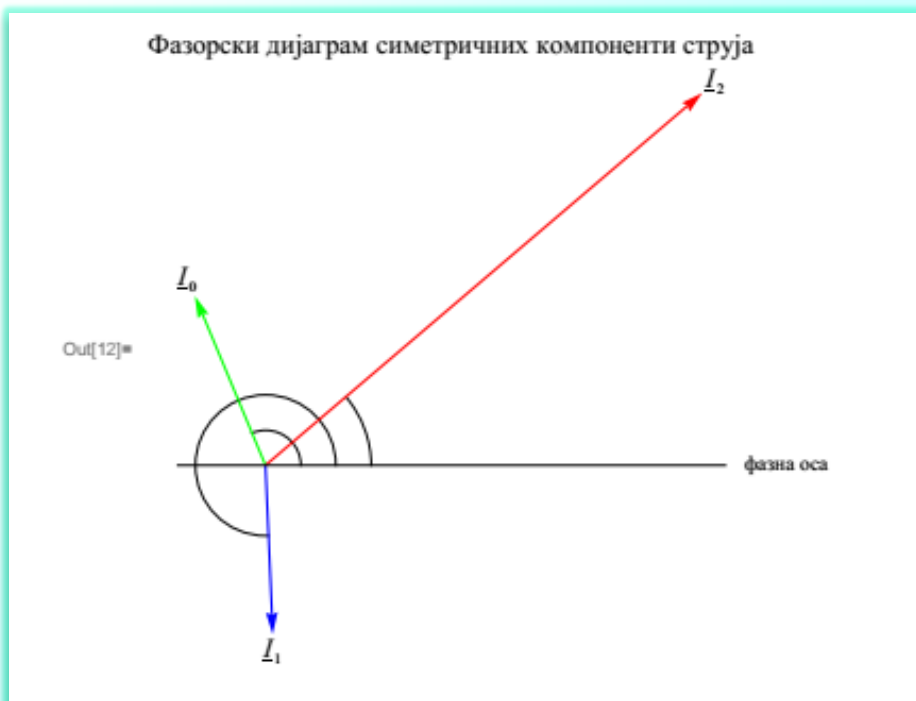
Учестаност је $f = 50 \text{ Hz}$.

Симетричне компоненте трофазног система струја

$$\mathbf{I}_{abc} = \begin{bmatrix} 150e^{j45^\circ} \\ 250e^{j150^\circ} \\ 100e^{j300^\circ} \end{bmatrix}$$

$$\underline{a} = e^{\frac{2\pi}{3}j}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$



$$\mathbf{I}_{012} = \mathbf{A}^{-1} \mathbf{I}_{abc} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 150e^{j45^\circ} \\ 250e^{j150^\circ} \\ 100e^{j300^\circ} \end{bmatrix}$$

$$\mathbf{I}_{012} = \begin{pmatrix} 52.1991 \angle 112.703^\circ \\ 48.0206 \angle -87.5867^\circ \\ 163.205 \angle 40.4521^\circ \end{pmatrix}$$

Тренутне вредности симетричних компоненти

$$\mathbf{I}_{012} = \begin{pmatrix} 52.1991 \angle 112.703^\circ \\ 48.0206 \angle -87.5867^\circ \\ 163.205 \angle 40.4521^\circ \end{pmatrix}$$

$$f = 50 \text{ Hz}$$

$$i_0(t) = 52.199\sqrt{2} \cos(2\pi ft + 112.703^\circ)$$

$$i_1(t) = 48.0206\sqrt{2} \cos(2\pi ft - 87.5867^\circ)$$

$$i_2(t) = 163.205\sqrt{2} \cos(2\pi ft + 40.4521^\circ)$$

Симетричне компоненте трофазног система струја

Iabc:=matrix([150*exp(I*45*PI/180), 250*exp(I*150*PI/180), 100*exp(I*300*PI/180)])

$$\begin{pmatrix} \sqrt{2} (75 + 75 i) \\ -125 \sqrt{3} + 125 i \\ 50 - 50 \sqrt{3} i \end{pmatrix}$$

a:=exp(I*2*PI/3)

$$-\frac{1}{2} + \frac{\sqrt{3} i}{2}$$

A:=matrix([[1,1,1], [1, a^(-1), a^(-2)], [1, a, a^2]])

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{1}{-\frac{1}{2} + \frac{\sqrt{3} i}{2}} & \frac{1}{\left(-\frac{1}{2} + \frac{\sqrt{3} i}{2}\right)^2} \\ 1 & -\frac{1}{2} + \frac{\sqrt{3} i}{2} & \left(-\frac{1}{2} + \frac{\sqrt{3} i}{2}\right)^2 \end{pmatrix}$$

I012:=simplify((A^(-1))*Iabc)

$$\begin{pmatrix} \sigma_1 + \sqrt{3} \left(-\frac{125}{3} - \frac{50}{3} i\right) + \frac{50}{3} + \frac{125}{3} i \\ \sigma_1 - \frac{100}{3} - \frac{250}{3} i \\ \sigma_1 + \sqrt{3} \left(\frac{125}{3} + \frac{50}{3} i\right) + \frac{50}{3} + \frac{125}{3} i \end{pmatrix}$$

where

$$\mathbf{I}_{012} = \mathbf{A}^{-1} \mathbf{I}_{abc}$$

$$\sigma_1 = \sqrt{2} (25 + 25 i)$$

I012_abs:=float(abs(I012))

$$\begin{pmatrix} 52.19911672 \\ 48.02058352 \\ 163.2052157 \end{pmatrix}$$

I012_arg:=matrix([float(arg((I012[i]))) \$ i=1..3])

$$\begin{pmatrix} 1.967044698 \\ -1.528676811 \\ 0.7060223216 \end{pmatrix}$$

I012_arg_deg:=matrix([float(180/PI*arg((I012[i]))) \$ i=1..3])

$$\begin{pmatrix} 112.7033593 \\ -87.58672953 \\ 40.45209927 \end{pmatrix}$$

`I012_re:=float(Re(I012))`

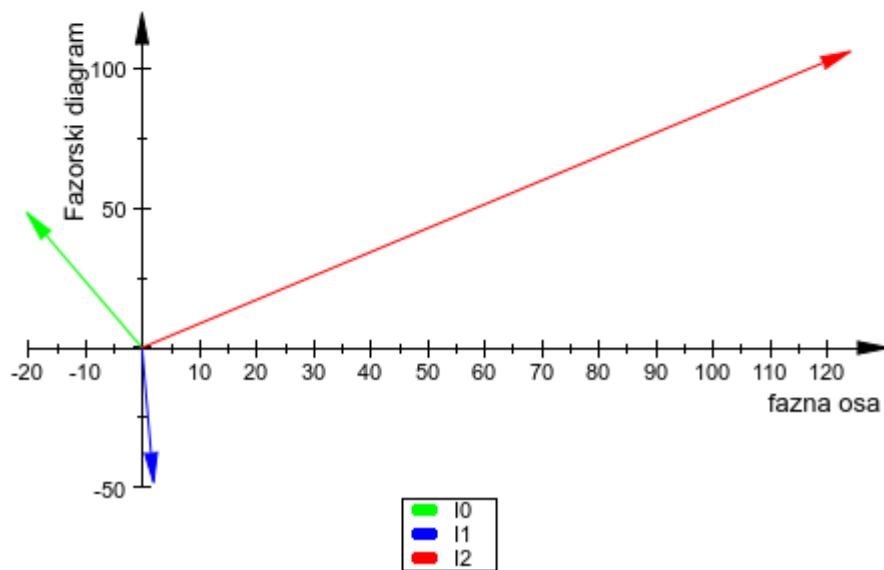
$$\begin{pmatrix} -20.14677792 \\ 2.022005726 \\ 124.1907894 \end{pmatrix}$$

`I012_im:=float(Im(I012))`

$$\begin{pmatrix} 48.15449227 \\ -47.97799427 \\ 105.8895192 \end{pmatrix}$$

Фазорски диаграм симетричних компоненти

```
plot(plot::Arrow2d([I012_re[1], I012_im[1]], Color = RGB::Green, Legend = "I0"),
plot::Arrow2d([I012_re[2], I012_im[2]], Legend = "I1"),
plot::Arrow2d([I012_re[3], I012_im[3]], Color = RGB::Red, Legend = "I2"),
Axes = Automatic,
AxesTitles= ["fazna osa", "Fazorski diagram"],
YAxisTitleOrientation = Vertical, YAxisTitleAlignment = End)
```



Тренутне вредности симетричних компоненти

f:=50

50

I012_arg[1]

1.967044698

I012_abs[1]

52.19911672

i0(t):=sqrt(2)*I012_abs[1]*cos(2*PI*f*t+I012_arg[1])

52.19911672 $\sqrt{2}$ cos(100 π t + 1.967044698)

i1(t):=sqrt(2)*I012_abs[2]*cos(2*PI*f*t+I012_arg[2])

48.02058352 $\sqrt{2}$ cos(100 π t - 1.528676811)

i2(t):=sqrt(2)*I012_abs[3]*cos(2*PI*f*t+I012_arg[3])

163.2052157 $\sqrt{2}$ cos(100 π t + 0.7060223216)

Задатак (2)

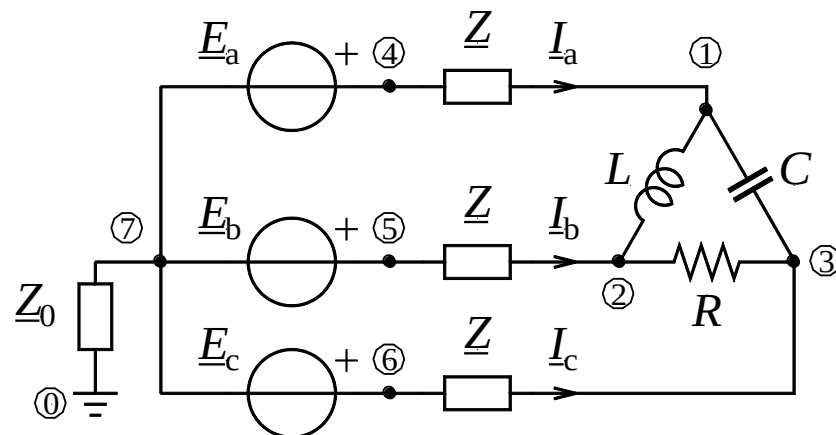
Вредности елемената трофазног електричног кола са слике су познате. Трофазни потрошач је повезан у троугао, а чине га отпорник, кондензатор и калем чији су параметри повезани изразима

$$C = \frac{1}{\sqrt{3}R\omega}, \quad L = \frac{\sqrt{3}R}{\omega}.$$

Симетричан трофазни генератор је повезан у звезду, а његови напони чине директан симетричан трофазни систем напона $\{e_a, e_b, e_c\}$ и $e_a(t) = \sqrt{2}E \cos(\omega t + \theta)$.

Одредити

- (5) симетричне компоненте трофазног система напона генератора,
- (5) симетричне компоненте трофазног система линијских струја, и
- (5) реактивну снагу потрошача.



Симетричне компоненте трофазног система напона генератора

$$C = \frac{1}{\sqrt{3}R\omega}$$

$$L = \frac{\sqrt{3}R}{\omega}$$

$$\begin{aligned}\underline{E}_a \\ \underline{E}_b &= \underline{E}_a e^{-\frac{2\pi}{3}j} \\ \underline{E}_c &= \underline{E}_a e^{-\frac{4\pi}{3}j}\end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$

$$\underline{a} = e^{\frac{2\pi}{3}j}$$

$$\mathbf{E}_{012} = \mathbf{A}^{-1} \begin{bmatrix} \underline{E}_a \\ \underline{E}_b \\ \underline{E}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \underline{E}_a \\ \underline{E}_b \\ \underline{E}_c \end{bmatrix}$$

$$\mathbf{E}_{012} = \begin{pmatrix} 0 \\ \underline{E}_a \\ 0 \end{pmatrix}$$

Симетричне компоненте трофазног система линијских струја

$$-\underline{I}_a + \frac{\underline{E}_a - \underline{V}_1}{\underline{Z}} = 0$$

$$-\underline{I}_b + \frac{\underline{E}_b - \underline{V}_2}{\underline{Z}} = 0$$

$$-\underline{I}_c + \frac{\underline{E}_c - \underline{V}_3}{\underline{Z}} = 0$$

$$\mathbf{I}_{abc} = \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix}$$

$$-\underline{I}_a + \frac{\underline{V}_1 - \underline{V}_2}{j\omega L} + j\omega C (\underline{V}_1 - \underline{V}_3) = 0$$

$$-\underline{I}_b + \frac{\underline{V}_2 - \underline{V}_1}{j\omega L} + \frac{\underline{V}_2 - \underline{V}_3}{R} = 0$$

$$-\underline{I}_c + \frac{\underline{V}_3 - \underline{V}_2}{R} + j\omega C (\underline{V}_3 - \underline{V}_1) = 0$$

$$\underline{I}_a = \frac{\underline{E}_a}{R + \underline{Z}}$$

$$\underline{I}_b = -\frac{(-1)^{1/3} \underline{E}_a}{R + \underline{Z}}$$

$$\underline{I}_c = \frac{j(j + \sqrt{3}) \underline{E}_a}{2(R + \underline{Z})}$$

$$\underline{V}_1 = \frac{\underline{E}_a R}{R + \underline{Z}}$$

$$\underline{V}_2 = -\frac{(-1)^{1/3} \underline{E}_a R}{R + \underline{Z}}$$

$$\underline{V}_3 = \frac{(-1)^{2/3} \underline{E}_a R}{R + \underline{Z}}$$

$$\mathbf{I}_{012} = \mathbf{A}^{-1} \mathbf{I}_{abc} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix}$$

$$\mathbf{I}_{012} = \begin{bmatrix} 0 \\ \frac{\underline{E}_a}{R + \underline{Z}} \\ 0 \end{bmatrix}$$

Реактивна снага потрошача

$$\underline{E}_a = E_a e^{j\theta}$$

$$\underline{Z} = R_{\text{lin}} + jX_{\text{lin}}$$

$$\underline{S} = \frac{|\underline{V}_1 - \underline{V}_2|^2}{(j\omega L)^*} + \frac{|\underline{V}_2 - \underline{V}_3|^2}{(R)^*} + \frac{|\underline{V}_3 - \underline{V}_1|^2}{\left(\frac{1}{j\omega C}\right)^*} = \frac{3E_a^2 R}{|R + R_{\text{lin}} + jX_{\text{lin}}|^2}$$

$$\underline{S} = P + jQ$$

$$P = \frac{3E_a^2 R}{(R + R_{\text{lin}})^2 + X_{\text{lin}}^2}$$

$$Q = 0$$

$$Q = 0$$

Симетричне компоненте трофазног система напона генератора

w:=`ω`

ω

Theta:=`θ`

θ

a:=exp(I*2*PI/3)

$$-\frac{1}{2} + \frac{\sqrt{3}i}{2}$$

assume(R>0 and w>0 and Z<>0 and Ea<>0 and
Eeff>0 and Rlin>0 and Xlin>0 and Theta>0)

C:=1/(R*w*sqrt(3))

$$\frac{\sqrt{3}}{3 \omega R}$$

L:=R*sqrt(3)/w

$$\frac{\sqrt{3} R}{\omega}$$

Eb:=Ea*a^(-1)

$$\frac{Ea}{-\frac{1}{2} + \frac{\sqrt{3}i}{2}}$$

Ec:=Ea*a^(-2)

$$\frac{Ea}{\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^2}$$

Eabc:=matrix([Ea, Eb, Ec])

$$\begin{pmatrix} Ea \\ \frac{Ea}{-\frac{1}{2} + \frac{\sqrt{3}i}{2}} \\ \frac{Ea}{\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^2} \end{pmatrix}$$

A:=matrix([[1,1,1], [1, a^(-1), a^(-2)], [1, a, a^2]])

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{1}{-\frac{1}{2} + \frac{\sqrt{3}i}{2}} & \frac{1}{\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^2} \\ 1 & -\frac{1}{2} + \frac{\sqrt{3}i}{2} & \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^2 \end{pmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$

E012:=simplify(A^(-1)*Eabc)

$$\begin{pmatrix} 0 \\ Ea \\ 0 \end{pmatrix}$$

$$\mathbf{E}_{012} = \mathbf{A}^{-1} \begin{bmatrix} \underline{E}_a \\ \underline{E}_b \\ \underline{E}_c \end{bmatrix}$$

jednachine:={-Ia+(Ea-V1)/Z=0,
 -Ib+(Eb-V2)/Z=0,
 -Ic+(Ec-V3)/Z=0,
 -Ia+(V1-V2)/(I*w*L)+I*w*C*(V1-V3)=0,
 -Ib+(V2-V1)/(I*w*L)+(V2-V3)/R=0,
 -Ic+(V3-V2)/R+I*w*C*(V3-V1)=0}

Једначине...

$$\left\{ -I_a - \sigma_2 + \sigma_1 = 0, -I_b - \frac{V_2 - \frac{E_a}{\sigma_3}}{Z} = 0, -I_c - \frac{V_3 - \frac{E_a}{\sigma_2}}{Z} = 0, \frac{V_2 - V_3}{R} - I_b + \sigma_2 = 0, -I_c - \frac{V_2 - V_3}{R} - \sigma_1 = 0, \frac{E_a - V_1}{Z} - I_a = 0 \right\}$$

where

$$\sigma_1 = \frac{\sqrt{3} (V_1 - V_3) i}{3 R}$$

$$\sigma_2 = \frac{\sqrt{3} (V_1 - V_2) i}{3 R}$$

$$\sigma_3 = -\frac{1}{2} + \frac{\sqrt{3} i}{2}$$

promenljive:={Ia, Ib, Ic, V1, V2, V3}

{Ia, Ib, Ic, V1, V2, V3}

odziv:=solve(jednacine, promenljive)

$$\left\{ \begin{aligned} & \left\{ \left[I_a = -\frac{E_a + 2z}{Z \sigma_9}, I_b = \frac{(E_a + 2z - 3\sqrt{3} E_a i - \sigma_1) i}{Z \sigma_9 (\sqrt{3} - i)}, I_c = \frac{2(2 E_a + z + \sqrt{3} z i)}{Z \sigma_9^2}, \right. \right. \\ & \quad \left. \left. V_1 = -\frac{(2z - 3 E_a + \sqrt{3} E_a i + \sigma_1) i}{\sigma_5}, V_2 = -\frac{(3 E_a + 2z - \sqrt{3} E_a i - \sigma_1) i}{\sigma_5}, V_3 = z \right] \right\} \end{aligned} \right. \quad \text{if } R + Z = 0$$

$$\left\{ \begin{aligned} & \left\{ \left[I_a = -\frac{2 E_a (6 R Z + \sigma_8 + \sigma_7 + 3 R^2 + Z^2 + 2\sqrt{3} R Z i)}{\sigma_2}, I_b = \frac{2 E_a \sigma_3}{\sigma_2}, I_c = \frac{4 E_a}{\sigma_4}, \right. \right. \\ & \quad \left. \left. V_1 = -\frac{2 E_a (R Z^2 + 6 R^2 Z + 3\sqrt{3} R^3 i + 3 R^3 + \sqrt{3} R Z^2 i + 2\sqrt{3} R^2 Z i)}{\sigma_2}, V_2 = \frac{2 E_a R \sigma_3}{\sigma_2}, V_3 = \frac{4 E_a R}{\sigma_4} \right] \right\} \end{aligned} \right. \quad \text{if } R^2 + 2 R Z + Z^2 \neq 0 \wedge 3 R^2 + 3 R Z + Z^2 \neq \sigma_6$$

where

$$\sigma_1 = 2 \sqrt{3} z i$$

$$\sigma_2 = (R + Z) \sigma_9^2 (3 R Z + 3 R^2 + Z^2 - \sigma_6)$$

$$\sigma_3 = \sigma_8 + \sigma_7 - 3 R^2 - Z^2 + 4 \sqrt{3} R Z i$$

$$\sigma_4 = (R + Z) \sigma_9^2$$

$$\sigma_5 = \sigma_9 (\sqrt{3} - i)$$

$$\sigma_6 = \sqrt{3} R Z i$$

$$\sigma_7 = \sqrt{3} Z^2 i$$

$$\sigma_8 = 3 \sqrt{3} R^2 i$$

$$\sigma_9 = -1 + \sqrt{3} i$$

Симетричне компоненте трофазног система линијских струја

Iabc:=Simplify(matrix([Ia | odziv[2], Ib | odziv[2], Ic | odziv[2]]))

$$\begin{pmatrix} \frac{E_a}{R + Z} \\ -\frac{E_a (1 + \sqrt{3} i)}{2 (R + Z)} \\ \frac{E_a (-1 + \sqrt{3} i)}{2 (R + Z)} \end{pmatrix}$$

$$\begin{aligned} \underline{I}_a &= \frac{\underline{E}_a}{R + \underline{Z}} \\ \underline{I}_b &= -\frac{(-1)^{1/3} \underline{E}_a}{R + \underline{Z}} \\ \underline{I}_c &= \frac{j(j + \sqrt{3}) \underline{E}_a}{2(R + \underline{Z})} \end{aligned}$$

$$\left[\text{rectform}((-1)^{(1/3)}) \right]$$

$$\frac{1}{2} + \frac{\sqrt{3}}{2} i$$

I012:=simplify(A^(-1)*Iabc)

$$\begin{pmatrix} 0 \\ \frac{E_a}{R + Z} \\ 0 \end{pmatrix}$$

$$\mathbf{I}_{012} = \mathbf{A}^{-1} \mathbf{I}_{abc}$$

Реактивна снага потрошача

$$U12:=\text{Simplify}((V1-V2) \mid \text{odziv}[2])$$

$$\frac{E_a R (3 + \sqrt{3} i)}{2 (R + Z)}$$

$$U23:=\text{Simplify}((V2-V3) \mid \text{odziv}[2])$$

$$-\frac{\sqrt{3} E_a R i}{R + Z}$$

$$U31:=\text{Simplify}((V3-V1) \mid \text{odziv}[2])$$

$$\frac{E_a R (-3 + \sqrt{3} i)}{2 (R + Z)}$$

$$S:=\text{Simplify}((\text{abs}(U12))^2/\text{conjugate}(I*w*L) + (\text{abs}(U23))^2/\text{conjugate}(R) + (\text{abs}(U31))^2/\text{conjugate}(1/(I*w*C)) \mid \{E_a=E_{\text{eff}}*\exp(I*\text{Theta}), Z=R_{\text{lin}}+I*X_{\text{lin}}\})$$

$$\frac{3 E_{\text{eff}}^2 R}{(R + R_{\text{lin}})^2 + X_{\text{lin}}^2}$$

$$P:=\text{Re}(S)$$

$$\frac{3 E_{\text{eff}}^2 R}{(R + R_{\text{lin}})^2 + X_{\text{lin}}^2}$$

$$Q:=\text{Im}(S)$$

$$0$$

$$\underline{S} = \frac{|\underline{V}_1 - \underline{V}_2|^2}{(j\omega L)^*} + \frac{|\underline{V}_2 - \underline{V}_3|^2}{(R)^*} + \frac{|\underline{V}_3 - \underline{V}_1|^2}{\left(\frac{1}{j\omega C}\right)^*} = \frac{3E_a^2 R}{|R + R_{\text{lin}} + jX_{\text{lin}}|^2}$$

$$\underline{S} = P + jQ$$

$$P = \frac{3E_a^2 R}{(R + R_{\text{lin}})^2 + X_{\text{lin}}^2}$$

$$Q = 0$$

Задатак (3)

У трофазном електричном колу је дошло до квара и одређене су симетричне компоненте трофазног система струја

$$\underline{I}_0 = 52 \text{ A} \angle 112^\circ, \underline{I}_1 = 48 \text{ A} \angle -88^\circ, \underline{I}_2 = 163 \text{ A} \angle 40^\circ.$$

- (a) Одредити трофазни систем струја.
- (б) Нацртати фазорски дијаграм трофазног система струја.
- (в) Одредити тренутне вредности трофазног система струја.

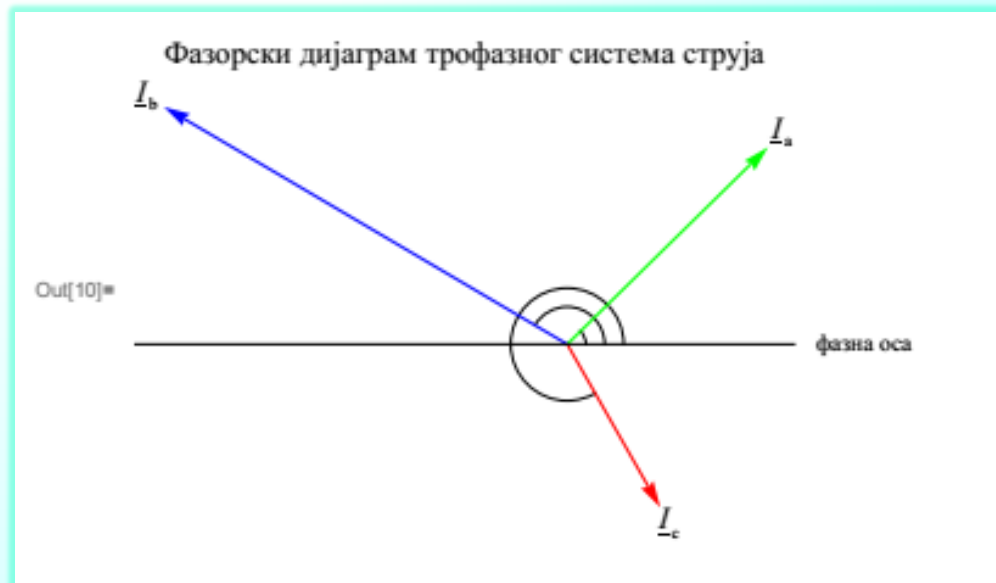
Учестаност је $f = 50 \text{ Hz}$.

Трофазни систем струја

$$\mathbf{I}_{012} = \begin{bmatrix} 52e^{j112^\circ} \\ 48e^{-j88^\circ} \\ 163e^{j40^\circ} \end{bmatrix}$$

$$\underline{a} = e^{\frac{2\pi}{3}j}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$



$$\mathbf{I}_{abc} = \mathbf{A} \cdot \mathbf{I}_{012} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \cdot \begin{bmatrix} 52e^{j112^\circ} \\ 48e^{-j88^\circ} \\ 163e^{j40^\circ} \end{bmatrix}$$

$$\mathbf{I}_{abc} = \begin{pmatrix} 149.969 \angle 44.4479^\circ \\ 249.479 \angle 149.533^\circ \\ 100.002 \angle -60.3103^\circ \end{pmatrix}$$

Тренутне вредности трофазног система струја

$$\mathbf{I}_{abc} = \begin{pmatrix} 149.969 \angle 44.4479^\circ \\ 249.479 \angle 149.533^\circ \\ 100.002 \angle -60.3103^\circ \end{pmatrix}$$

$$f = 50 \text{ Hz}$$

$$i_a(t) = 149.969\sqrt{2} \cos(2\pi ft + 44.4479^\circ)$$

$$i_b(t) = 249.479\sqrt{2} \cos(2\pi ft + 149.533^\circ)$$

$$i_c(t) = 100.002\sqrt{2} \cos(2\pi ft - 60.3103^\circ)$$

Трофазни систем струја

```
I012:=matrix([52*exp(I*112*PI/180), 48*exp(-I*88*PI/180), 163*exp(I*40*PI/180)])
```

$$\begin{pmatrix} 52 e^{\frac{28 \pi i}{45}} \\ -48 e^{\frac{23 \pi i}{45}} \\ 163 e^{\frac{2 \pi i}{9}} \end{pmatrix}$$

$$\mathbf{I}_{abc} = \mathbf{A} \cdot \mathbf{I}_{012}$$

```
a:=exp(I*2*PI/3)
```

$$-\frac{1}{2} + \frac{\sqrt{3} i}{2}$$

```
A:=matrix([[1,1,1], [1, a^(-1), a^(-2)], [1, a, a^2]])
```

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{1}{-\frac{1}{2} + \frac{\sqrt{3} i}{2}} & \frac{1}{\left(-\frac{1}{2} + \frac{\sqrt{3} i}{2}\right)^2} \\ 1 & -\frac{1}{2} + \frac{\sqrt{3} i}{2} & \left(-\frac{1}{2} + \frac{\sqrt{3} i}{2}\right)^2 \end{pmatrix}$$

```
Iabc:=simplify(A*I012)
```

$$\begin{pmatrix} \sigma_1 \left(13 \sqrt{5} - 48 \sigma_3 + 150 + 13 \sqrt{2} \sqrt{\sqrt{5} + 5 i} \right) \\ \frac{\sigma_1 (48 \sigma_3 + 26 \sqrt{5} + 26 \sqrt{2} \sqrt{\sqrt{5} + 5 i} - 189 + 48 \sqrt{3} \sigma_3 i + 163 \sqrt{3} i)}{2} \\ 52 e^{\frac{28 \pi i}{45}} - 48 e^{\frac{23 \pi i}{45}} \sigma_2 + 163 \sigma_1 \sigma_2^2 \end{pmatrix}$$

where

$$\sigma_1 = e^{\frac{2 \pi i}{9}}$$

$$\sigma_2 = -\frac{1}{2} + \frac{\sqrt{3} i}{2}$$

$$\sigma_3 = e^{\frac{13 \pi i}{45}}$$

```
Iabc_abs:=float(abs(Iabc))
```

$$\begin{pmatrix} 149.968796 \\ 249.479279 \\ 100.002248 \end{pmatrix}$$

```
Iabc_arg:=matrix([float(arg((Iabc[i]))) $ i=1..3])
```

$$\begin{pmatrix} 0.7757619339 \\ 2.609838529 \\ -1.05261268 \end{pmatrix}$$

Фазорски дијаграм

```
Iabc_arg_deg:=matrix([float(180/PI*arg((Iabc[i]))) $ i=1..3])
```

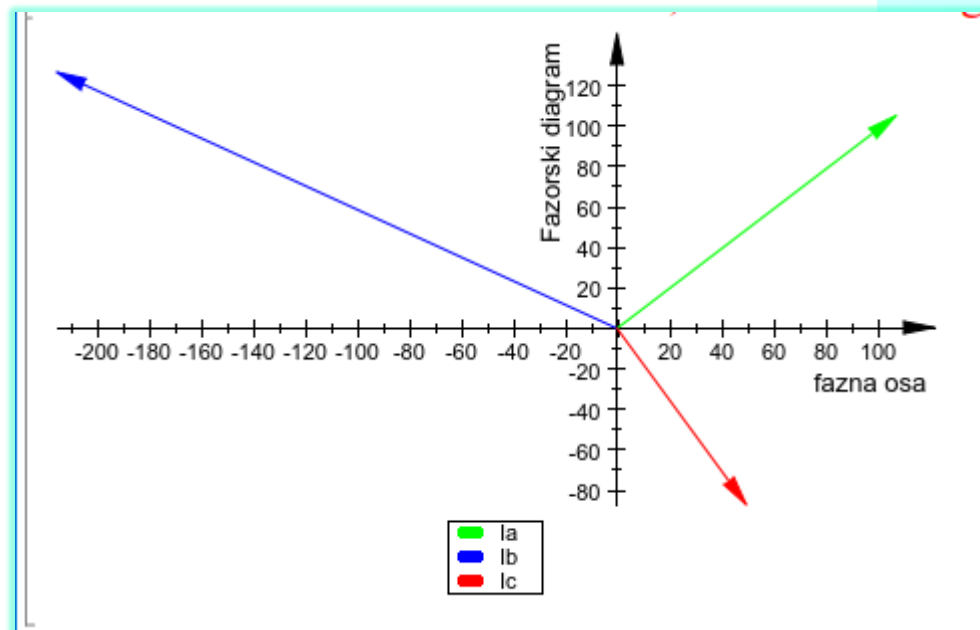
```
( 44.44788472  
149.5327329  
-60.31026403 )
```

```
Iabc_re:=float(Re(Iabc))
```

```
( 107.0608772  
-215.0309245  
49.53141872 )
```

```
Iabc_im:=float(Im(Iabc))
```

```
( 105.0171811  
126.4974788  
-86.87397862 )
```



```
plot(plot::Arrow2d([Iabc_re[1], Iabc_im[1]], Color = RGB::Green, Legend = "Ia"),  
plot::Arrow2d([Iabc_re[2], Iabc_im[2]], Legend = "Ib"),  
plot::Arrow2d([Iabc_re[3], Iabc_im[3]], Color = RGB::Red, Legend = "Ic"),  
Axes = Automatic,  
AxesTitles= ["fazna osa", "Fazorski diagram"],  
YAxisTitleOrientation = Vertical, YAxisTitleAlignment = End)
```

Тренутне вредности трофазног система струја

f:=50

50

ia(t):=sqrt(2)*Iabc_abs[1]*cos(2*PI*f*t+Iabc_arg[1])

149.968796 $\sqrt{2}$ cos(100 π t + 0.7757619339)

ib(t):=sqrt(2)*Iabc_abs[2]*cos(2*PI*f*t+Iabc_arg[2])

249.479279 $\sqrt{2}$ cos(100 π t + 2.609838529)

ic(t):=sqrt(2)*Iabc_abs[3]*cos(2*PI*f*t+Iabc_arg[3])

100.002248 $\sqrt{2}$ cos(100 π t - 1.05261268)

Задатак (4)

Вредности елемената трофазног електричног кола са слике су познате.

Трофазни потрошач је повезан у звезду, а чине га три спрегнута калема сопствених индуктивности L и међусобних индуктивности M .

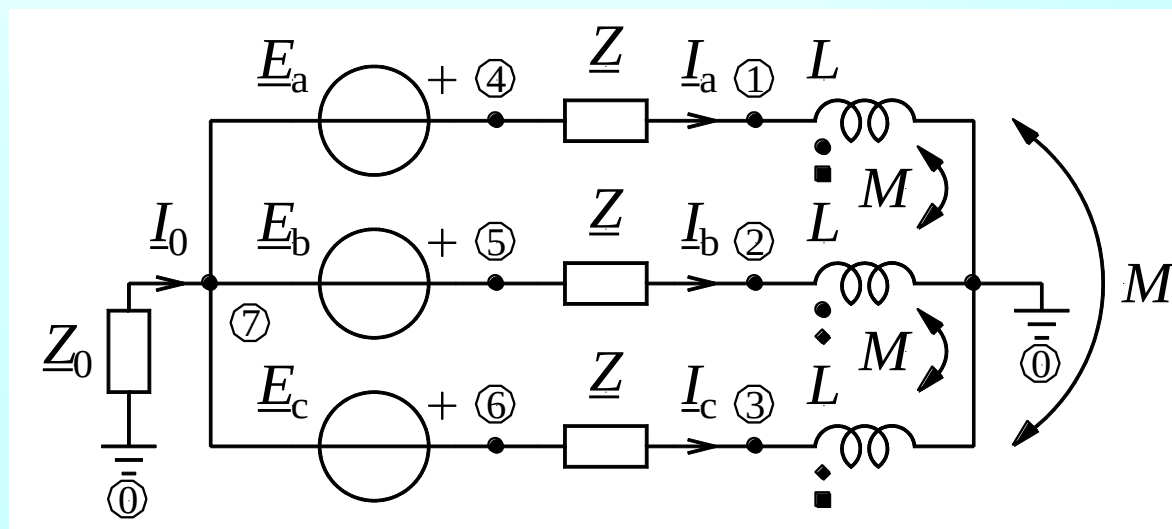
Симетричан трофазни генератор је повезан у звезду, а његови напони чине инверзан симетричан трофазни систем напона $\{e_a, e_b, e_c\}$ и $e_a = \sqrt{2}E \cos(\omega t + \theta)$.

Одредити

(5) симетричне компоненте трофазног система напона генератора,

(5) симетричне компоненте трофазног система линијских струја, и

(5) симетричне компоненте импеданси потрошача.



Симетричне компоненте трофазног система напона генератора

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$

$$\underline{a} = e^{\frac{2\pi}{3}j}$$

$$\underline{E}_a$$

$$\underline{E}_b = \underline{E}_a e^{\frac{2\pi}{3}j}$$

$$\underline{E}_c = \underline{E}_a e^{\frac{4\pi}{3}j}$$

$$\mathbf{E}_{012} = \mathbf{A}^{-1} \begin{bmatrix} \underline{E}_a \\ \underline{E}_b \\ \underline{E}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \underline{E}_a \\ \underline{E}_b \\ \underline{E}_c \end{bmatrix}$$

$$\mathbf{E}_{012} = \begin{pmatrix} 0 \\ 0 \\ \underline{E}_a \end{pmatrix}$$

Једначине... Линијске струје...

$$\underline{I}_0 = \underline{I}_a + \underline{I}_b + \underline{I}_c$$

$$\underline{E}_a = \underline{Z}\underline{I}_a + \underline{U}_1 + \underline{Z}_0\underline{I}_0$$

$$\underline{E}_b = \underline{Z}\underline{I}_b + \underline{U}_2 + \underline{Z}_0\underline{I}_0$$

$$\underline{E}_c = \underline{Z}\underline{I}_c + \underline{U}_3 + \underline{Z}_0\underline{I}_0$$

$$\underline{U}_1 = j\omega L\underline{I}_a + j\omega M\underline{I}_b + j\omega M\underline{I}_c$$

$$\underline{U}_2 = j\omega M\underline{I}_a + j\omega L\underline{I}_b + j\omega M\underline{I}_c$$

$$\underline{U}_3 = j\omega M\underline{I}_a + j\omega M\underline{I}_b + j\omega L\underline{I}_c$$



$$\underline{I}_0 = 0$$

$$\underline{I}_a = \frac{\underline{E}_a}{\underline{Z} + j(L - M)\omega}$$

$$\underline{I}_b = \frac{j(j + \sqrt{3})\underline{E}_a}{2(\underline{Z} + j(L - M)\omega)}$$

$$\underline{I}_c = -\frac{(-1)^{1/3}\underline{E}_a}{\underline{Z} + j(L - M)\omega}$$

$$\underline{U}_1 = \frac{\underline{E}_a(L - M)\omega}{-j\underline{Z} + (L - M)\omega}$$

$$\underline{U}_2 = \frac{j(j + \sqrt{3})\underline{E}_a(L - M)\omega}{-2j\underline{Z} + 2(L - M)\omega}$$

$$\underline{U}_3 = -\frac{(-1)^{1/3}\underline{E}_a(L - M)\omega}{-j\underline{Z} + (L - M)\omega}$$

Симетричне компоненте трофазног система линијских струја

$$\mathbf{I}_{abc} = \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix} = \begin{bmatrix} \frac{\underline{E}_a}{\underline{Z} + j(L-M)\omega} \\ \frac{j(j + \sqrt{3})\underline{E}_a}{2(\underline{Z} + j(L-M)\omega)} \\ -\frac{(-1)^{1/3}\underline{E}_a}{\underline{Z} + j(L-M)\omega} \end{bmatrix}$$

$$\mathbf{I}_{012} = \mathbf{A}^{-1}\mathbf{I}_{abc} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{-1} & \underline{a}^{-2} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix}$$

$$\mathbf{I}_{012} = \begin{pmatrix} 0 \\ 0 \\ \frac{\underline{E}_a}{i(L-M)\omega + \underline{Z}} \end{pmatrix}$$

Симетричне компоненте импеданси потрошача

$$\mathbf{Z} = \begin{bmatrix} j\omega L & j\omega M & j\omega M \\ j\omega M & j\omega L & j\omega M \\ j\omega M & j\omega M & j\omega L \end{bmatrix}$$

$$\mathbf{Z}_s = \mathbf{A}^{-1} \mathbf{Z} \mathbf{A}$$

$$\mathbf{Z}_s = \begin{pmatrix} i(L + 2M)\omega & 0 & 0 \\ 0 & i(L - M)\omega & 0 \\ 0 & 0 & i(L - M)\omega \end{pmatrix}$$

Симетричне компоненте трофазног система напона генератора

w:='ω'

ω

a:=exp(I*2*PI/3)

$$-\frac{1}{2} + \frac{\sqrt{3}i}{2}$$

pobuda:={Eb=Ea*a, Ec=Ea*a^2}

$$\left\{ E_b = E_a \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right), E_c = E_a \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right)^2 \right\}$$

Eabc:=matrix([Ea, Eb, Ec]) | pobuda

$$\begin{pmatrix} E_a \\ E_a \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right) \\ E_a \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right)^2 \end{pmatrix}$$

A:=matrix([[1,1,1], [1, a^(-1), a^(-2)], [1, a, a^2]])

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \frac{1}{-\frac{1}{2} + \frac{\sqrt{3}i}{2}} & \frac{1}{\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right)^2} \\ 1 & -\frac{1}{2} + \frac{\sqrt{3}i}{2} & \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2} \right)^2 \end{pmatrix}$$

$$\mathbf{E}_{012} = \mathbf{A}^{-1} \begin{bmatrix} \underline{E}_a \\ \underline{E}_b \\ \underline{E}_c \end{bmatrix}$$

E012:=simplify(A^(-1)*Eabc)

$$\begin{pmatrix} 0 \\ 0 \\ E_a \end{pmatrix}$$

Једначине...

```
jednacine:={I0=Ia+Ib+Ic,  
Ea=Z*Ia+U1+Z0*I0,  
Eb=Z*Ib+U2+Z0*I0,  
Ec=Z*Ic+U3+Z0*I0,  
U1=I*w*L*Ia+I*w*M*Ib+I*w*M*Ic,  
U2=I*w*M*Ia+I*w*L*Ib+I*w*M*Ic,  
U3=I*w*M*Ia+I*w*M*Ib+I*w*L*Ic} | pobuda
```

$$\{Ea \sigma_1^2 = U3 + I0 Z0 + Ic Z, Ea = U1 + I0 Z0 + Ia Z, U1 = \omega Ia L i + \omega Ib M i + \omega Ic M i, U2 = \omega Ib L i + \omega Ia M i + \omega Ic M i, \\ U3 = \omega Ic L i + \omega Ia M i + \omega Ib M i, Ea \sigma_1 = U2 + I0 Z0 + Ib Z, I0 = Ia + Ib + Ic\}$$

where

$$\sigma_1 = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$$

```
promenljive:={I0, Ia, Ib, Ic, U1, U2, U3}  
{I0, Ia, Ib, Ic, U1, U2, U3}
```

Трофазни систем линијских струја

```
odziv:=linsolve(jednachine, promenljive, ShowAssumptions)
```

$$\left[\begin{aligned} &I_0 = 0, I_a = \frac{E_a}{\sigma_2}, I_b = -\frac{\sqrt{3} E_a + E_a i}{2 \sigma_1}, I_c = -\frac{-\sqrt{3} E_a + E_a i}{2 \sigma_1}, U_1 = -\frac{\omega E_a (L-M)}{\sigma_1}, U_2 = \frac{\omega E_a (L-M-\sqrt{3} L i + \sqrt{3} M i)}{2 \sigma_1}, U_3 = -\frac{\omega E_a (\sqrt{3} M - \sqrt{3} L - M i + L i)}{2 \sigma_2} \end{aligned} \right], [],$$

$$\left[\begin{aligned} &\omega (L^2 - M^2) i \neq 0, -\omega M i \neq 0, M Z_0 - L Z_0 - M Z - L Z \neq 0, -Z (M Z - L Z_0 + M Z_0) i \\ &\neq 0, -\omega^2 L^2 Z i - i Z_0 \omega^2 L^2 + 2 i Z_0 \omega^2 L M + \omega^2 M^2 Z i - i Z_0 \omega^2 M^2 - 2 \omega L Z^2 - 4 Z_0 \omega L Z + 4 Z_0 \omega M Z + Z^3 i + 3 i Z_0 Z^2 \\ &\neq 0, \omega^2 L^2 i + \omega^2 L M i - 2 \omega^2 M^2 i + 2 \omega L Z + 3 Z_0 \omega L + \omega M Z - 3 Z_0 \omega M - Z^2 i - 3 i Z_0 Z \neq 0 \end{aligned} \right]$$

where

$$\sigma_1 = \omega M - \omega L + Z i$$

$$\sigma_2 = Z + \omega L i - \omega M i$$

```
Simplify(I0 | odziv[1])
```

0

$$\underline{I}_0 = 0$$

Симетричне компоненте трофазног система линијских струја

$I_{abc} := \text{matrix}([\text{Simplify}(I_a | \text{odziv}[1]), \text{Simplify}(I_b | \text{odziv}[1]), \text{Simplify}(I_c | \text{odziv}[1])])$

$$\begin{pmatrix} \frac{E_a}{Z + \omega (L i - M i)} \\ -\frac{E_a \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i \right)}{-\omega (L - M) + Z i} \\ \frac{E_a \left(\frac{\sqrt{3}}{2} - \frac{1}{2} i \right)}{-\omega (L - M) + Z i} \end{pmatrix}$$

$$\begin{aligned} \underline{I}_a &= \frac{\underline{E}_a}{\underline{Z} + j(L - M)\omega} \\ \underline{I}_b &= \frac{j(j + \sqrt{3})\underline{E}_a}{2(\underline{Z} + j(L - M)\omega)} \\ \underline{I}_c &= -\frac{(-1)^{1/3} \underline{E}_a}{\underline{Z} + j(L - M)\omega} \end{aligned}$$

$$\begin{aligned} &\text{rectform}((-1)^{(1/3)}) \\ &\frac{1}{2} + \frac{\sqrt{3}}{2} i \end{aligned}$$

$U_{p123} := \text{matrix}([\text{Simplify}(U_1 | \text{odziv}[1]), \text{Simplify}(U_2 | \text{odziv}[1]), \text{Simplify}(U_3 | \text{odziv}[1])])$

$$\begin{pmatrix} -\frac{\omega E_a (L - M)}{-\omega (L - M) + Z i} \\ -\frac{\omega E_a (-1 + \sqrt{3} i) (L - M)}{2 (\omega M - \omega L + Z i)} \\ \frac{\omega E_a (L - M) (\sqrt{3} - i)}{2 (Z + \omega L i - \omega M i)} \end{pmatrix}$$

$$\begin{aligned} \underline{U}_1 &= \frac{\underline{E}_a (L - M)\omega}{-j\underline{Z} + (L - M)\omega} \\ \underline{U}_2 &= \frac{j(j + \sqrt{3})\underline{E}_a (L - M)\omega}{-2j\underline{Z} + 2(L - M)\omega} \\ \underline{U}_3 &= -\frac{(-1)^{1/3} \underline{E}_a (L - M)\omega}{-j\underline{Z} + (L - M)\omega} \end{aligned}$$

$I_{012} := \text{Simplify}(A^{(-1)} * I_{abc})$

$$\begin{pmatrix} 0 \\ 0 \\ \frac{E_a}{Z + \omega (L i - M i)} \end{pmatrix}$$

$$\mathbf{I}_{012} = \mathbf{A}^{-1} \mathbf{I}_{abc}$$

Симетричне компоненте импеданси потрошача

```
Z:=matrix([[I*w*L, I*w*M, I*w*M], [I*w*M, I*w*L, I*w*M], [I*w*M, I*w*M, I*w*L]])
```

$$\begin{pmatrix} \omega L i & \omega M i & \omega M i \\ \omega M i & \omega L i & \omega M i \\ \omega M i & \omega M i & \omega L i \end{pmatrix}$$

```
Zs:=Simplify(A^(-1)*Z*A)
```

$$\begin{pmatrix} \omega (L + 2 M) i & 0 & 0 \\ 0 & \omega (L - M) i & 0 \\ 0 & 0 & \omega (L - M) i \end{pmatrix}$$

$$\mathbf{Z}_s = \mathbf{A}^{-1} \mathbf{Z} \mathbf{A}$$