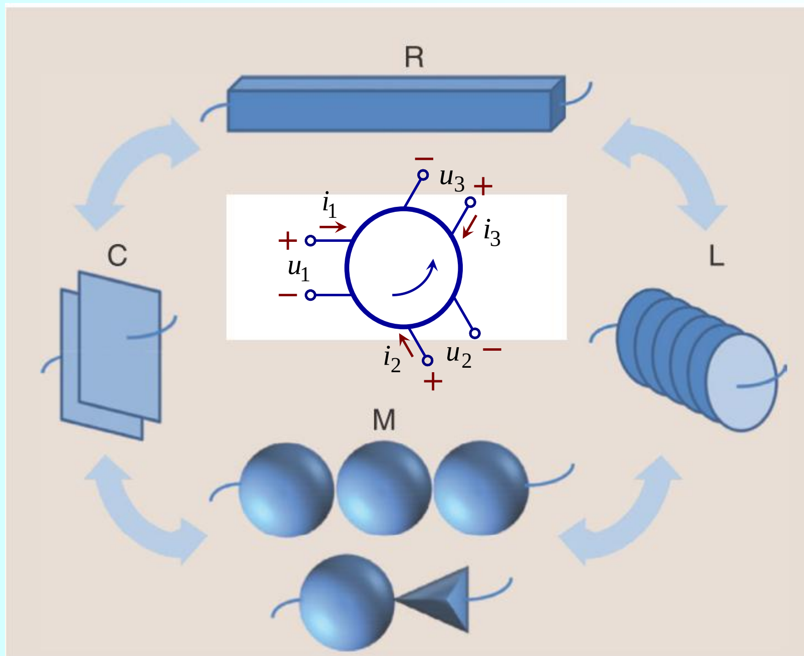
Практикум из рачунарске анализе кола

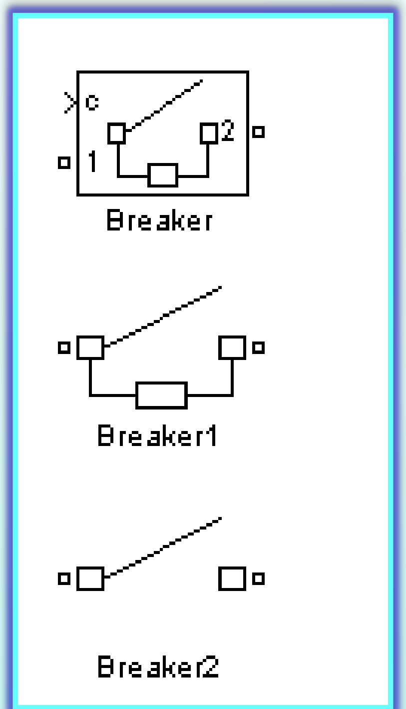


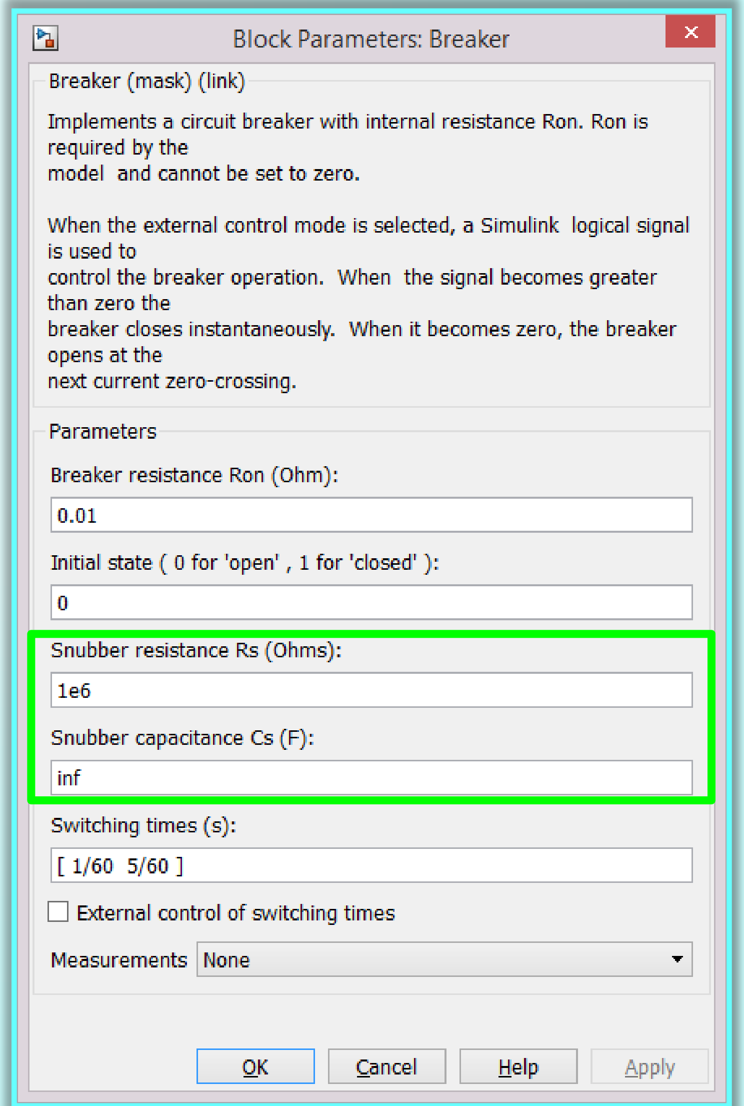
Милка Потребић

Прекидач, диода, трансформатор...

Модел **прекидача** (нелинеарни елемент)

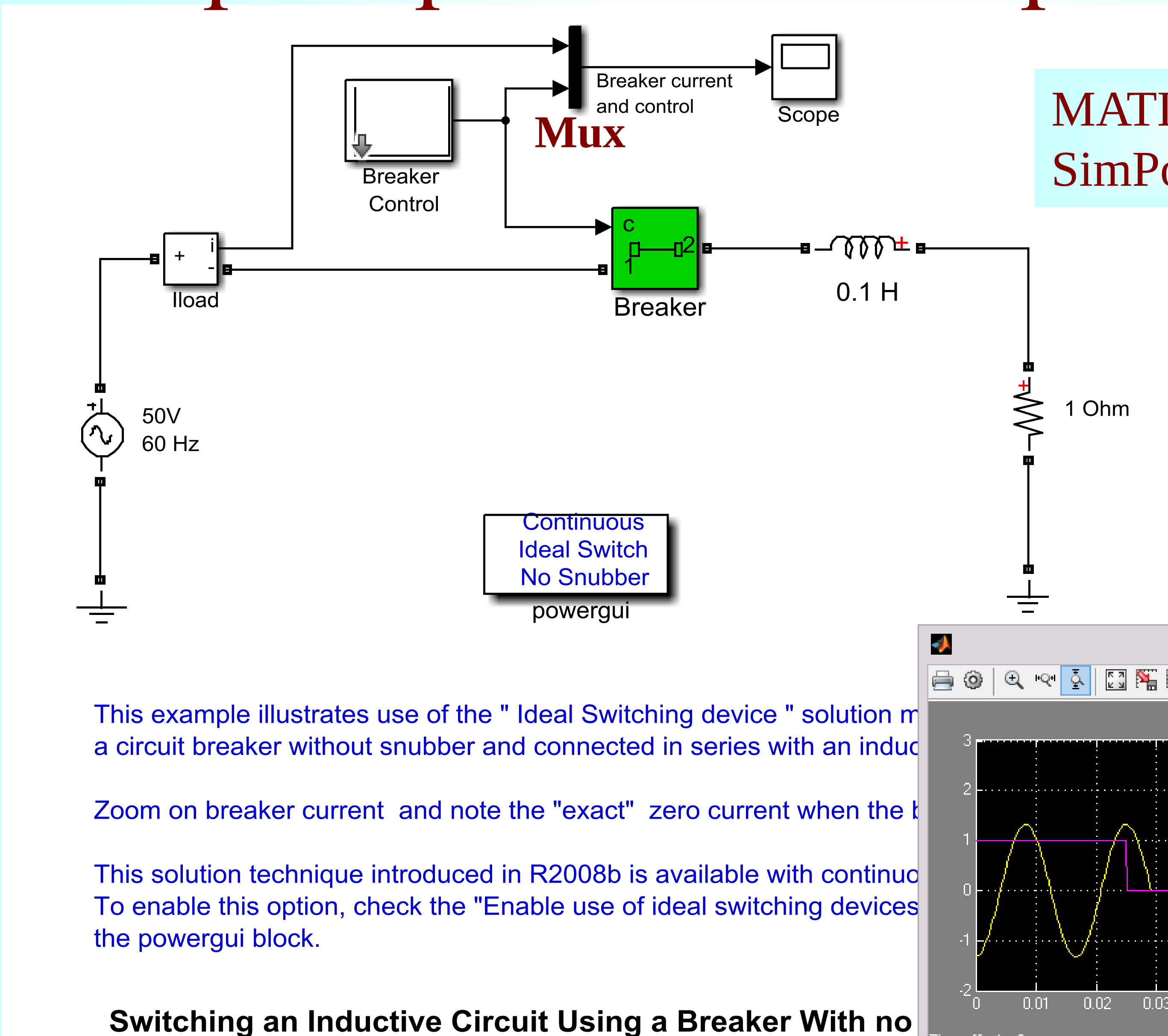
MATLAB: Simscape
SimPowerSystems





A series Rs-Cs snubber circuit is included in the model. It can be connected to the circuit breaker. If the Breaker block happens to be in series with an inductive circuit, an open circuit or a current source, you must use a snubber.

Пример ел. кола са прекидачем

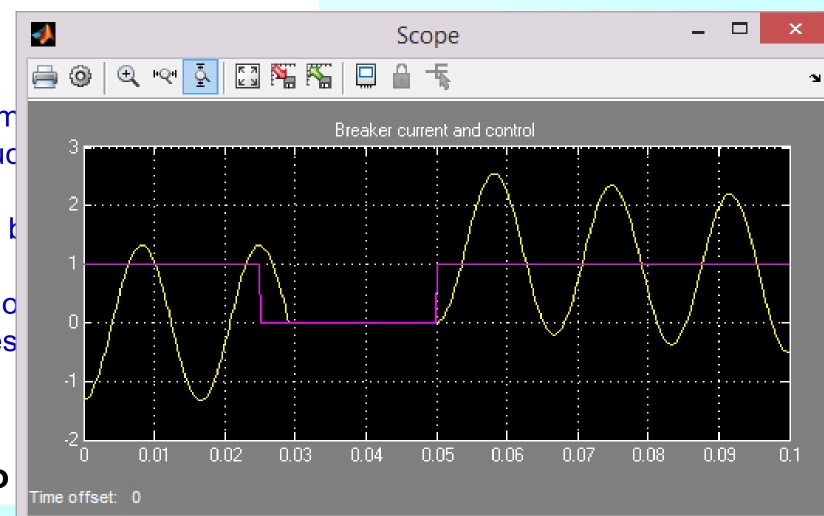


MATLAB: Simscape

SimPowerSystems

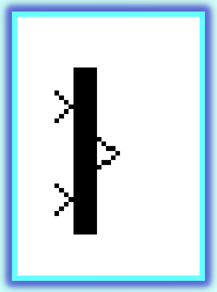
ode23tb

ode15s

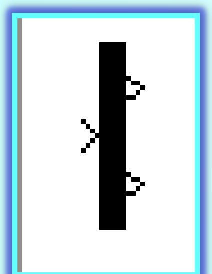


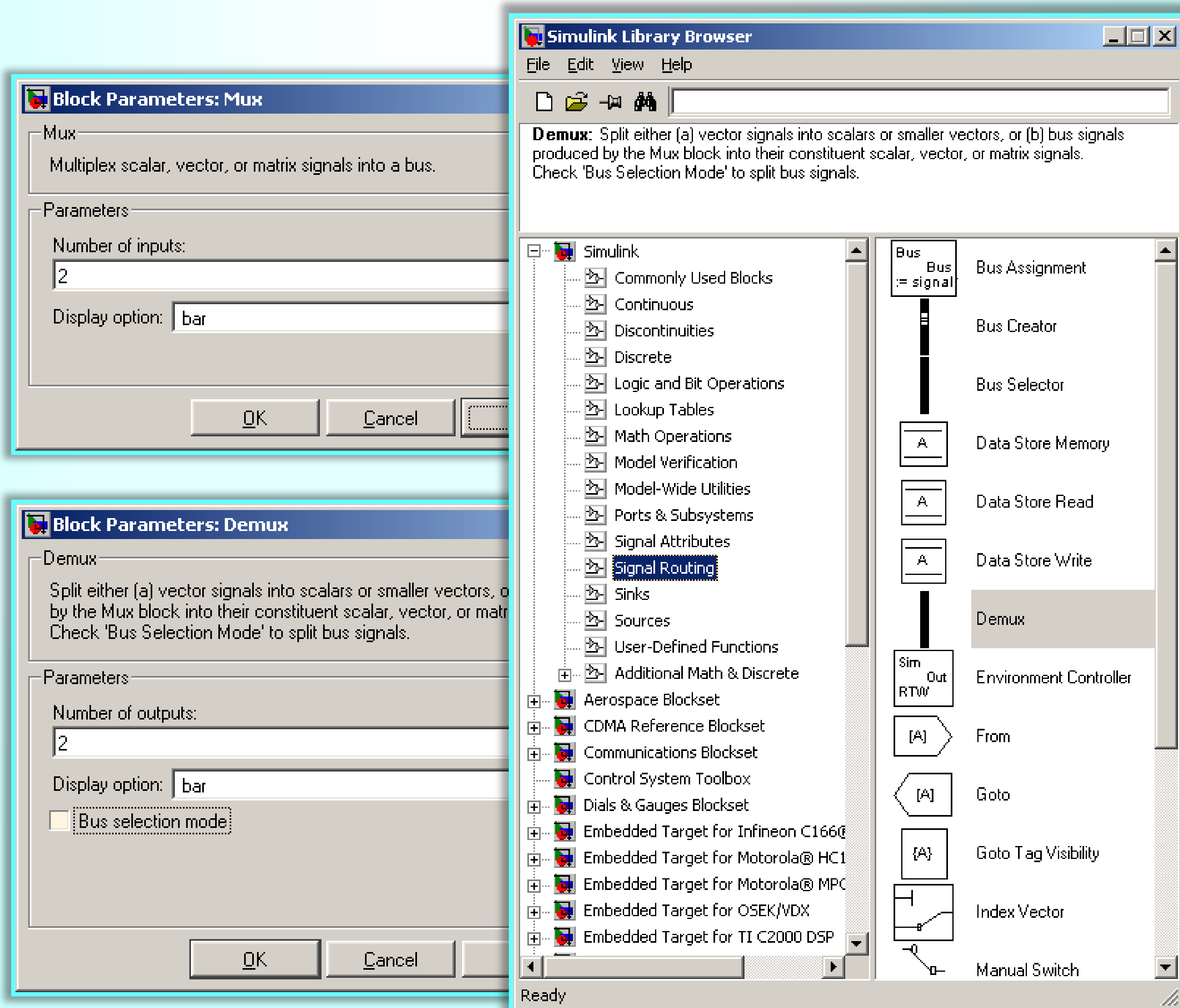
Мултиплексер/демултиплексер

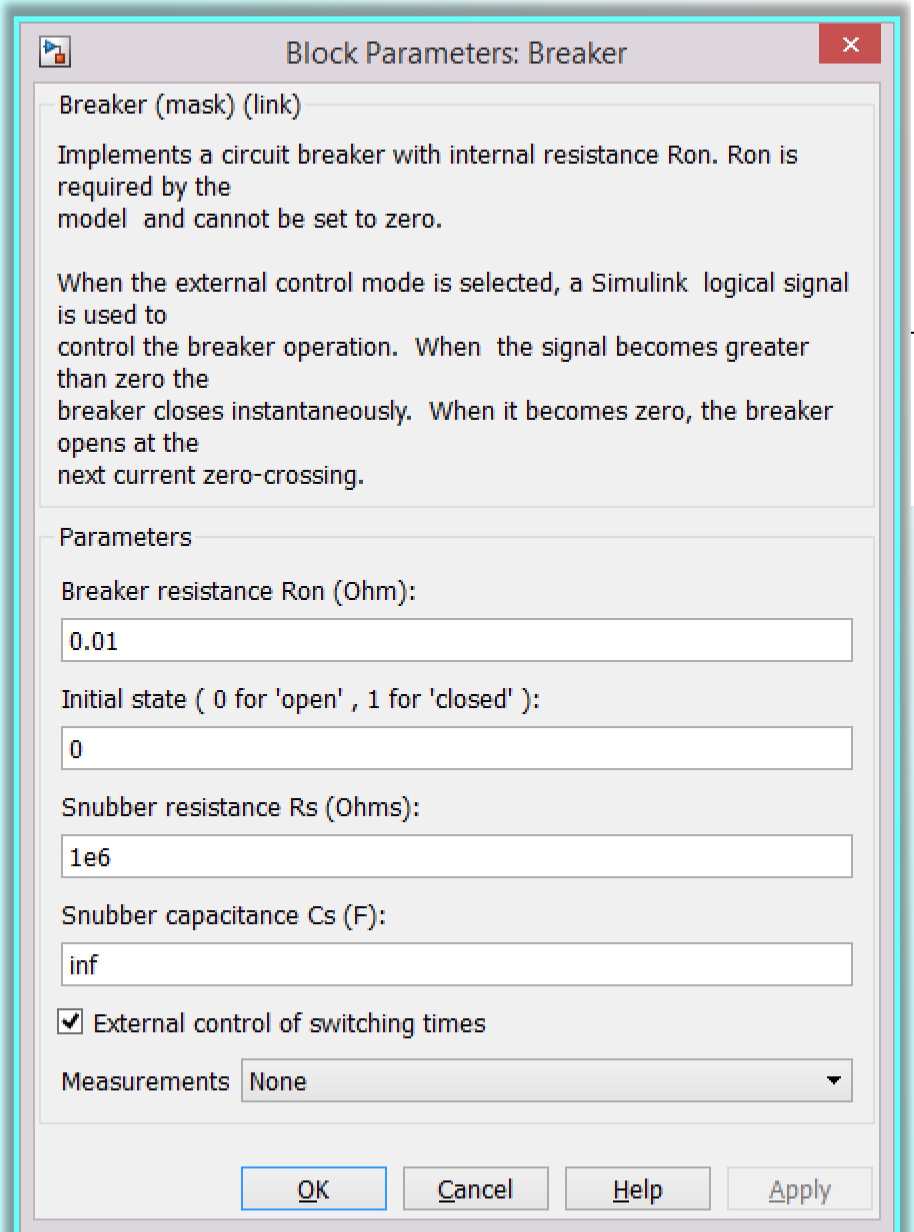
Mux

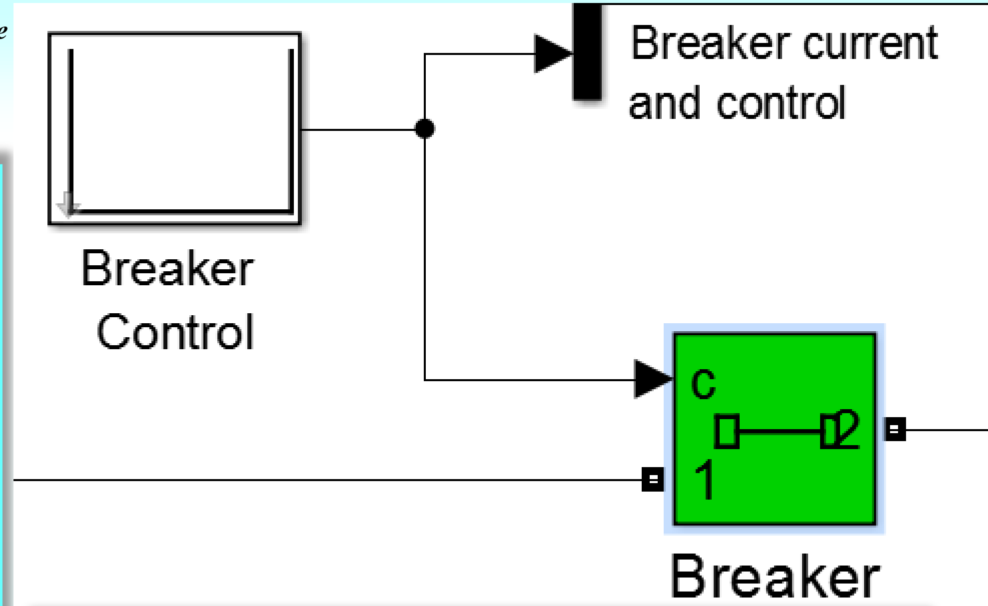


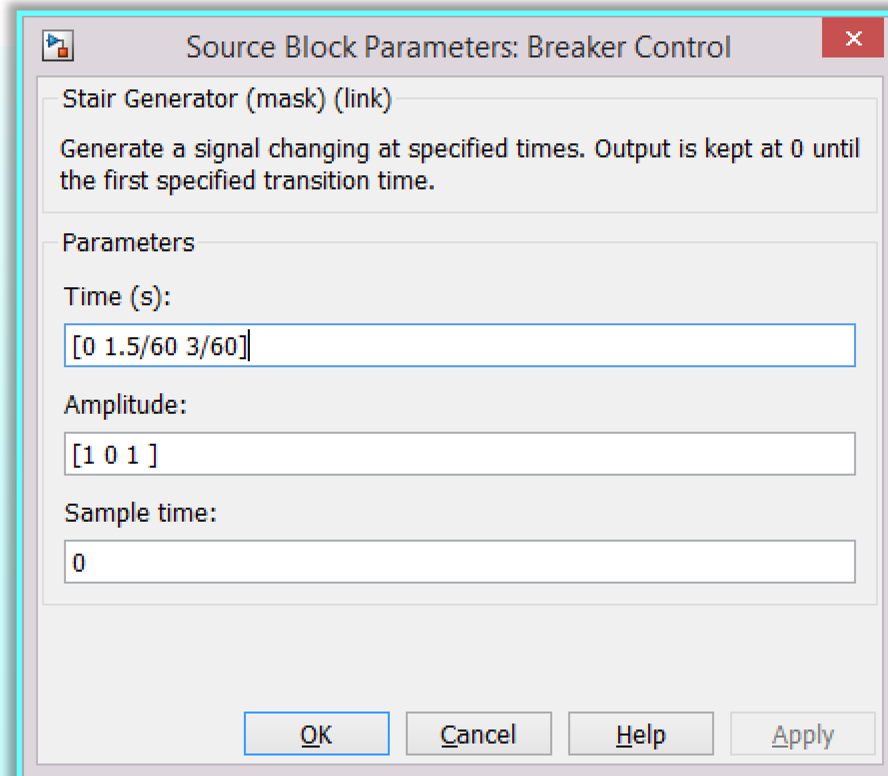
Demux





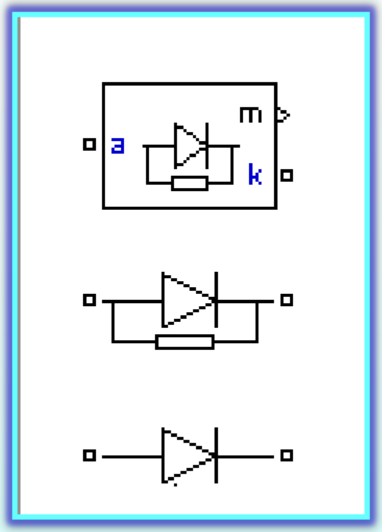

Block Parameters: Breaker
Breaker (mask) (link)
Implements a circuit breaker with internal resistance Ron. Ron is required by the model and cannot be set to zero.
When the external control mode is selected, a Simulink logical signal is used to control the breaker operation. When the signal becomes greater than zero the breaker closes instantaneously. When it becomes zero, the breaker opens at the next current zero-crossing.
Parameters
Breaker resistance Ron (Ohm):
0.01
Initial state (0 for 'open' , 1 for 'closed'):
0
Snubber resistance Rs (Ohms):
1e6
Snubber capacitance Cs (F):
inf
External control of switching times
Measurements
None
OK
Cancel
Help
Apply

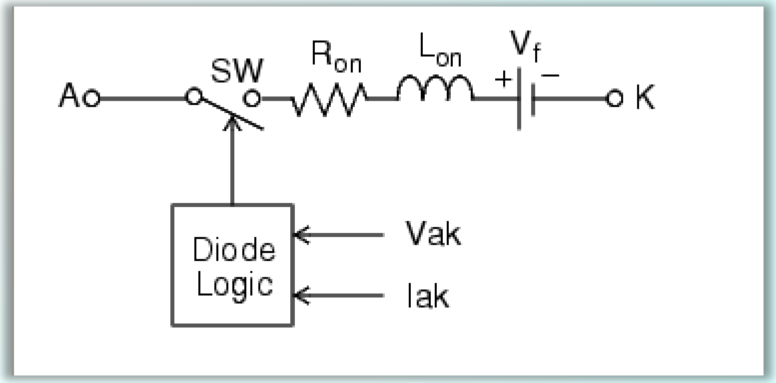

Breaker current and control
Breaker Control
c
2
1
Breaker

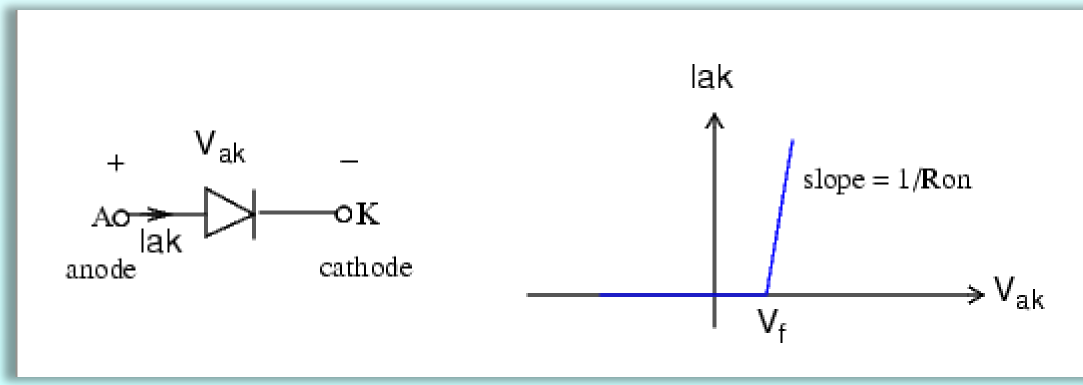

Source Block Parameters: Breaker Control
Stair Generator (mask) (link)
Generate a signal changing at specified times. Output is kept at 0 until the first specified transition time.
Parameters
Time (s):
[0 1.5/60 3/60]
Amplitude:
[1 0 1]
Sample time:
0
OK
Cancel
Help
Apply

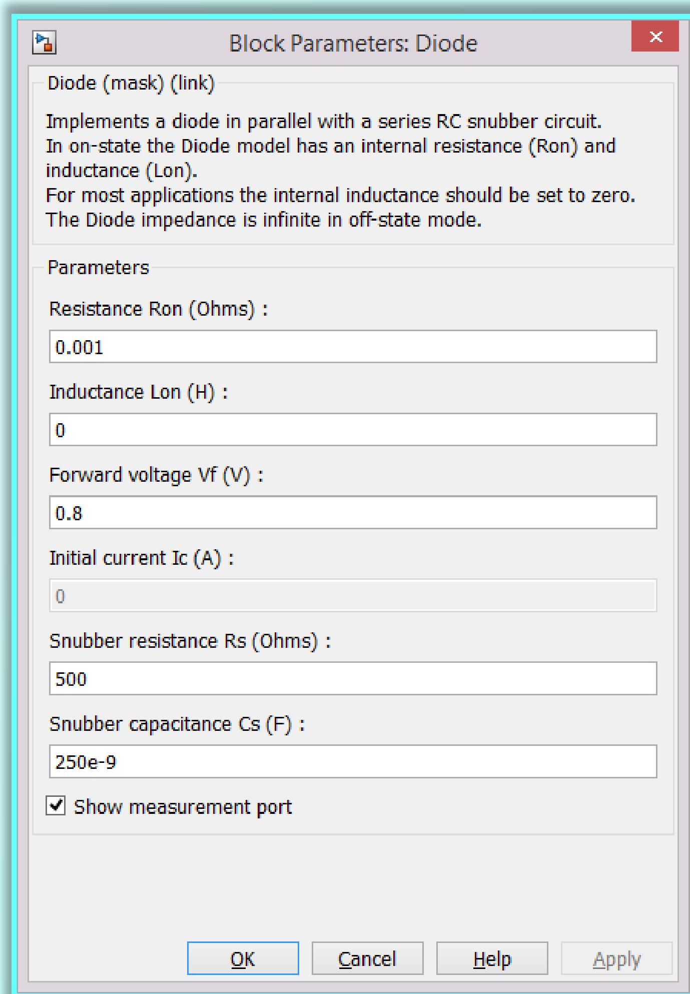
Модел **диоде** (нелинеарни елемент)

MATLAB: Simscape
SimPowerSystems

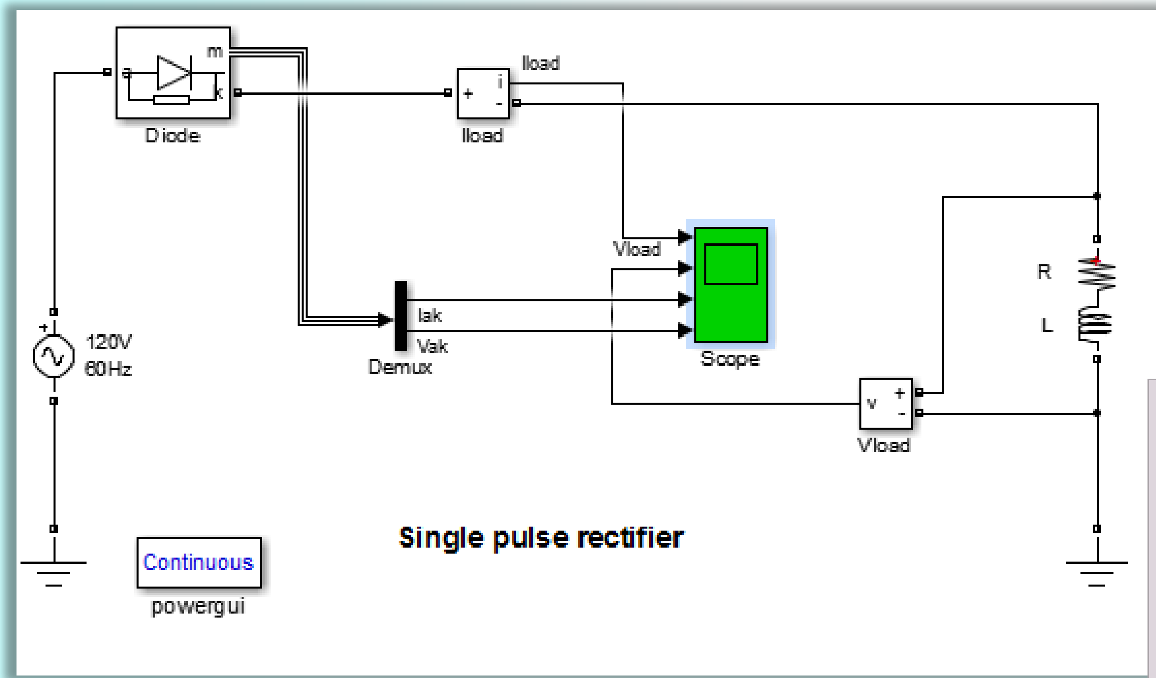




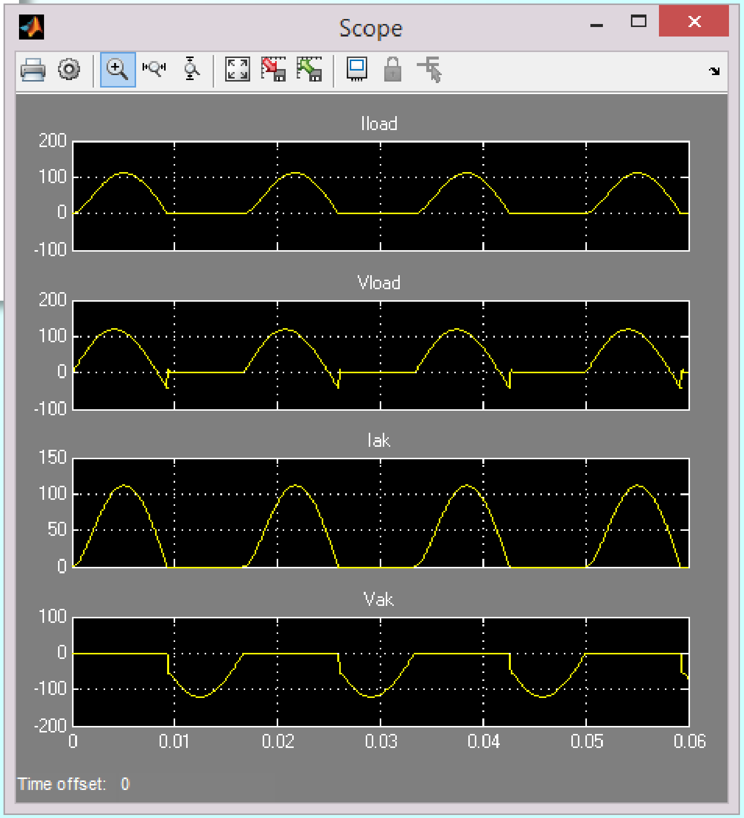




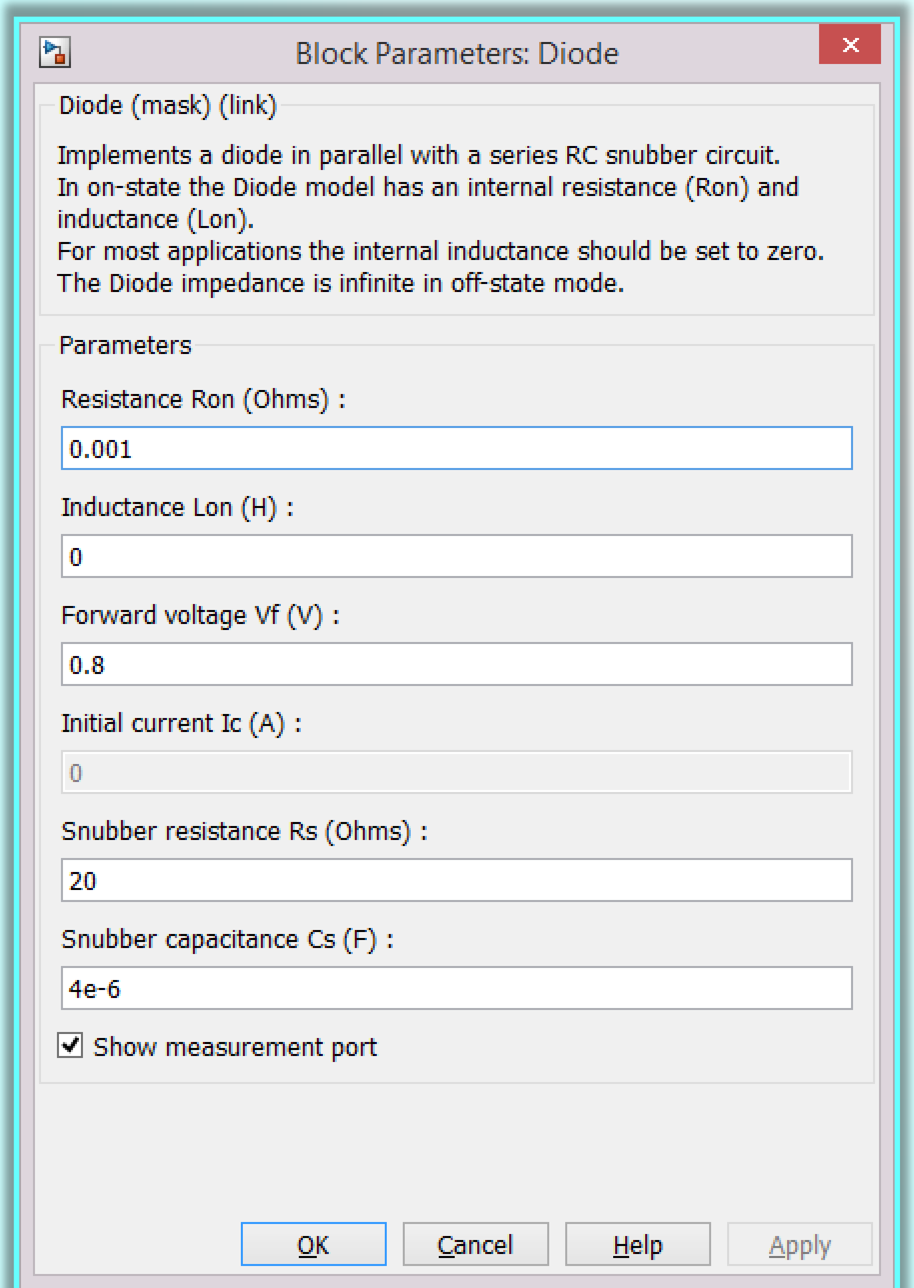
Пример ел. кола са диодом

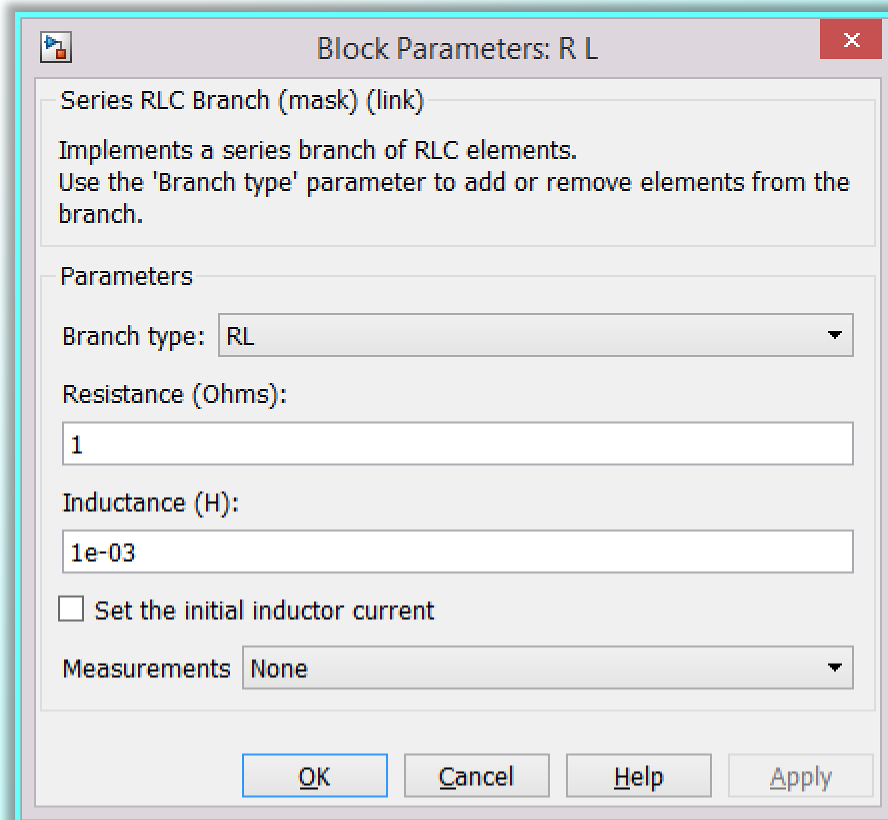


MATLAB: Simscape
SimPowerSystems



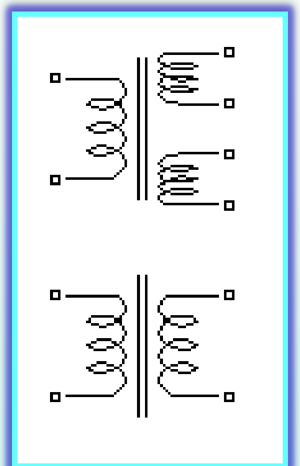
ode23tb
ode15s

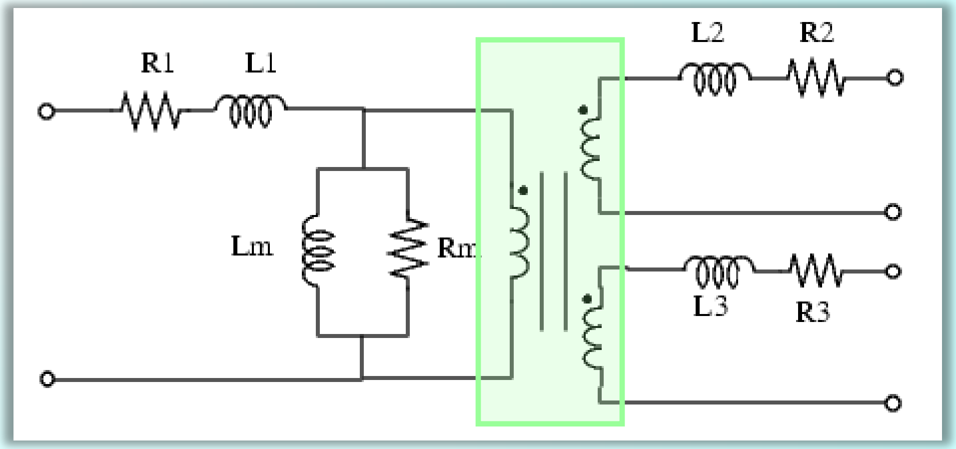

Block Parameters: Diode
Diode (mask) (link)
Implements a diode in parallel with a series RC snubber circuit.
In on-state the Diode model has an internal resistance (Ron) and inductance (Lon).
For most applications the internal inductance should be set to zero.
The Diode impedance is infinite in off-state mode.
Parameters
Resistance Ron (Ohms) :
0.001
Inductance Lon (H) :
0
Forward voltage Vf (V) :
0.8
Initial current Ic (A) :
0
Snubber resistance Rs (Ohms) :
20
Snubber capacitance Cs (F) :
4e-6
Show measurement port
OK
Cancel
Help
Apply


Block Parameters: R L
Series RLC Branch (mask) (link)
Implements a series branch of RLC elements.
Use the 'Branch type' parameter to add or remove elements from the branch.
Parameters
Branch type:
RL
Resistance (Ohms):
1
Inductance (H):
1e-03
Set the initial inductor current
Measurements
None
OK
Cancel
Help
Apply

Линеарни индуктивни трансформатор

- Модел ЛИТ за Енергетику са нормализованим вредностима





Идеални трансформатор

MATLAB: Simscape
SimPowerSystems

$$R_{base} = \frac{(Vn)^2}{Pn}$$

$$L_{base} = \frac{R_{base}}{2\pi fn}$$

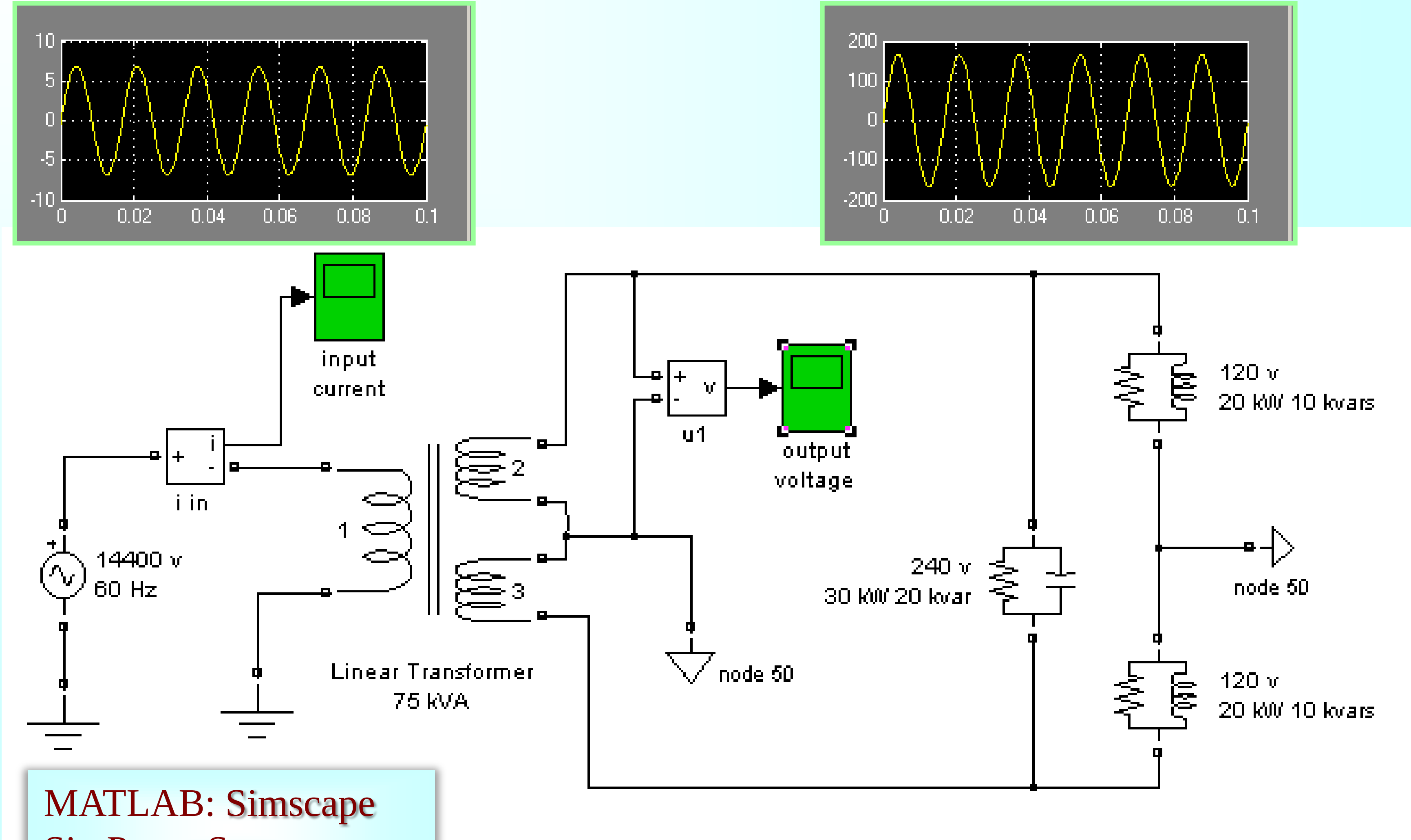
$$R(\text{p.u.}) = \frac{R(\Omega)}{R_{base}}$$

$$L(\text{p.u.}) = \frac{L(H)}{L_{base}}$$

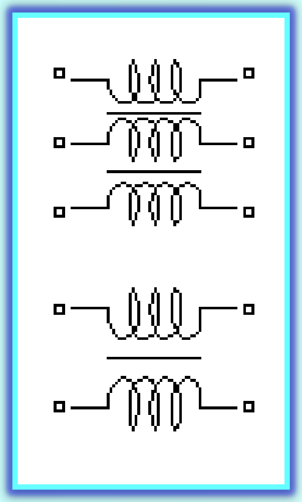
Modeling an Ideal Transformer

To implement an ideal transformer model, set the winding resistances and inductances to 0, and the magnetization resistance and inductance (Rm Lm) to `inf`.

Пример ел. кола са линеарним индуктивним трансформатором

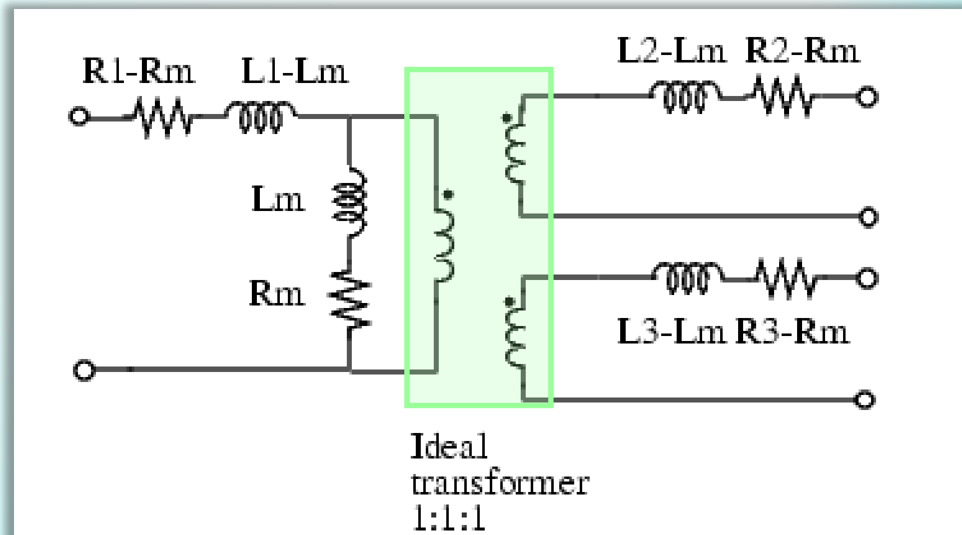


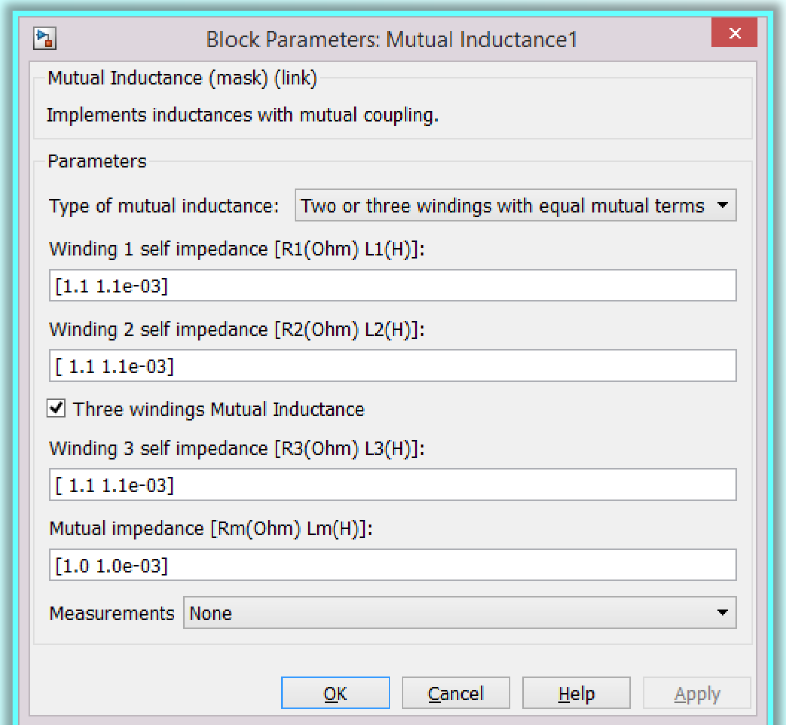
MATLAB: Simscape
SimPowerSystems



Mutual Inductance

- Основни модел ЛИТ за Енергетику

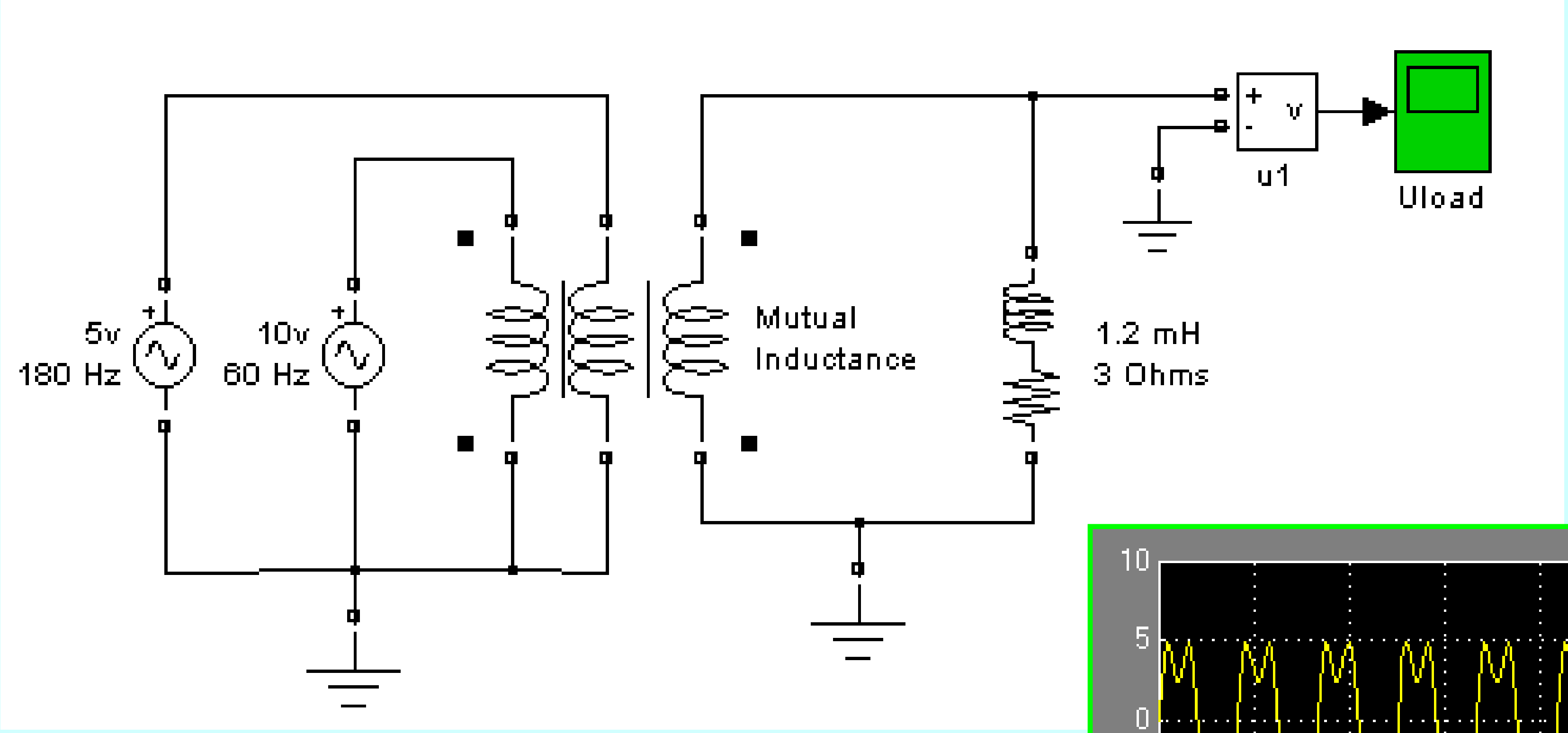


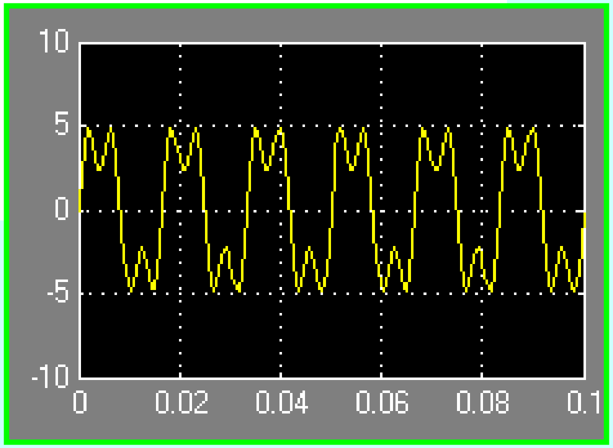


$$R_1, R_2, R_3 \neq R_m$$
$$L_1, L_2, L_3 \neq L_m$$

MATLAB: Simscape
SimPowerSystems

Пример ел. кола са моделом ЛИТ Mutual Inductance

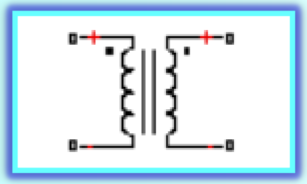




MATLAB: Simscape
SimPowerSystems

Mutual Inductance

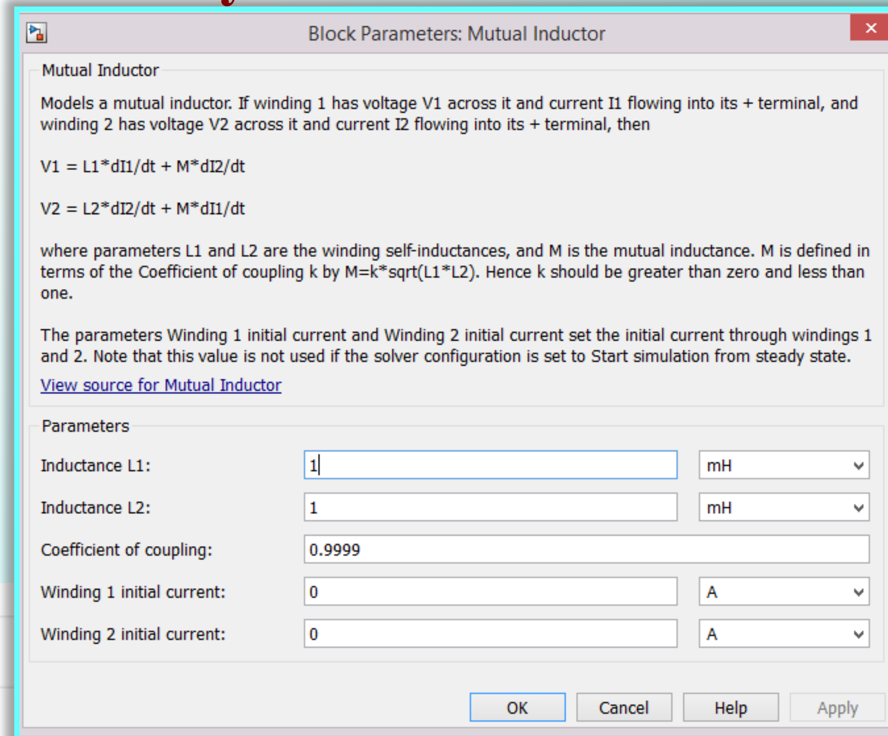
- Основни модел ЛИТ без губитака



$$V1 = L1\frac{dI1}{dt} + M\frac{dI2}{dt}$$

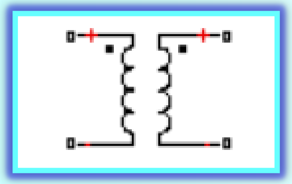
$$V2 = L2\frac{dI2}{dt} + M\frac{dI1}{dt}$$

$$M = k\sqrt{L1 \cdot L2}$$



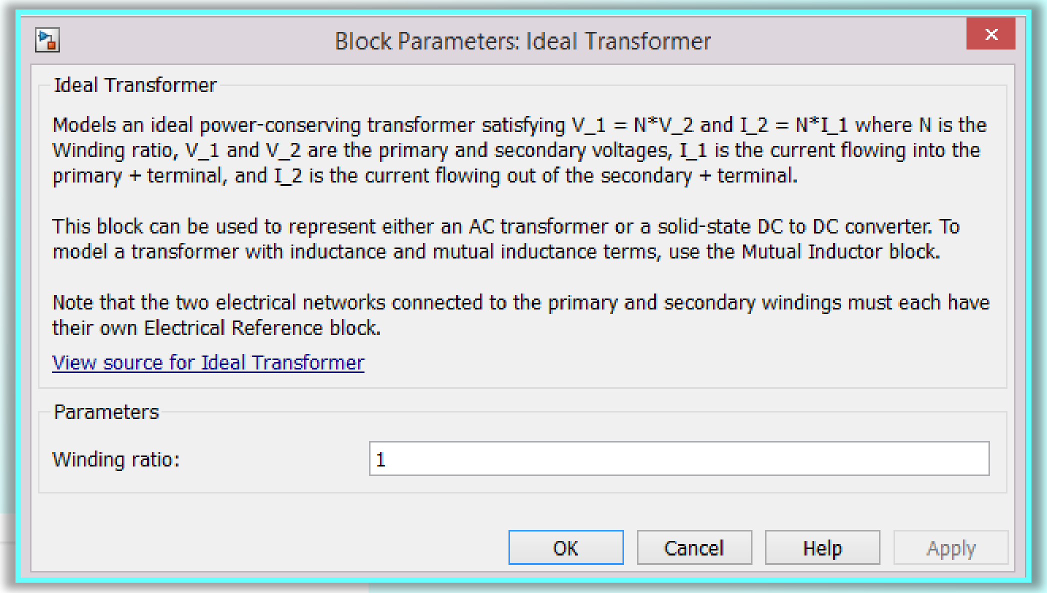
$V1$	Voltage across winding 1
$V2$	Voltage across winding 2
$I1$	Current flowing into the + terminal of winding 1
$I2$	Current flowing into the + terminal of winding 2
$L1$, $L2$	Winding self-inductances
M	Mutual inductance
k	Coefficient of coupling, $0 < k < 1$
t	Time

MATLAB: Simscape Foundation Library



Ideal Transformer

- Модел идеалног трансформатора



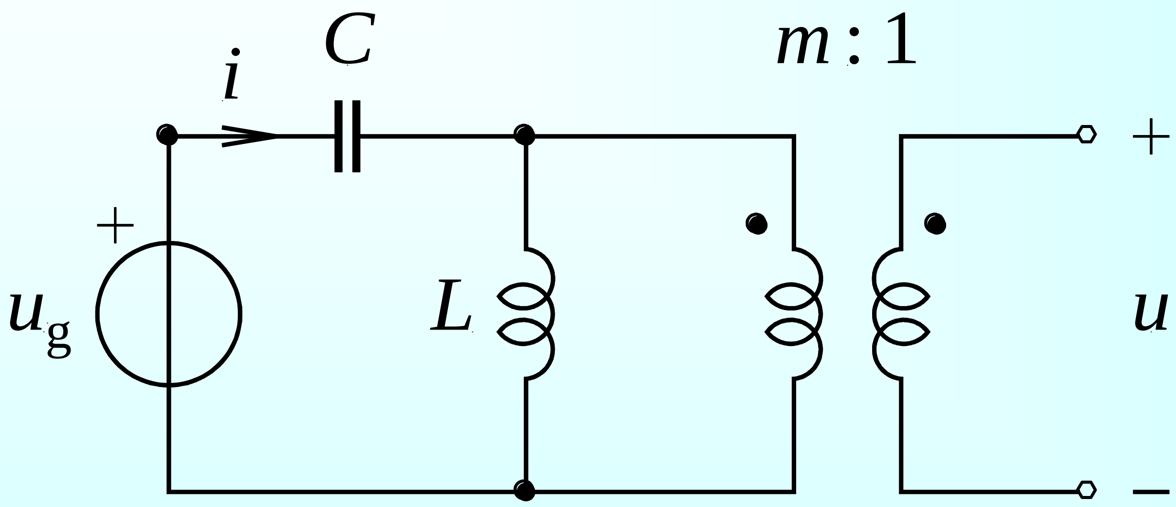
$$V1 = N \cdot V2$$

$$I2 = N \cdot I1$$

V1	Primary voltage
V2	Secondary voltage
I1	Current flowing into the primary + terminal
I2	Current flowing out of the secondary + terminal
N	Winding ratio

MATLAB: Simscape Foundation Library

Одзив на простопериодичну побуду



$$U_m = 1\,\mathrm{V}$$
$$\omega = 10^5\,\mathrm{rad/s}$$
$$L = 0.1\,\mathrm{mH}$$
$$C = 1\,\mu\mathrm{F}$$
$$m = 1$$

$$u_g(t) = U_m \sin\left(\frac{1}{\sqrt{CL}}t\right)h(t)$$

Задатак - простопериодична побуда

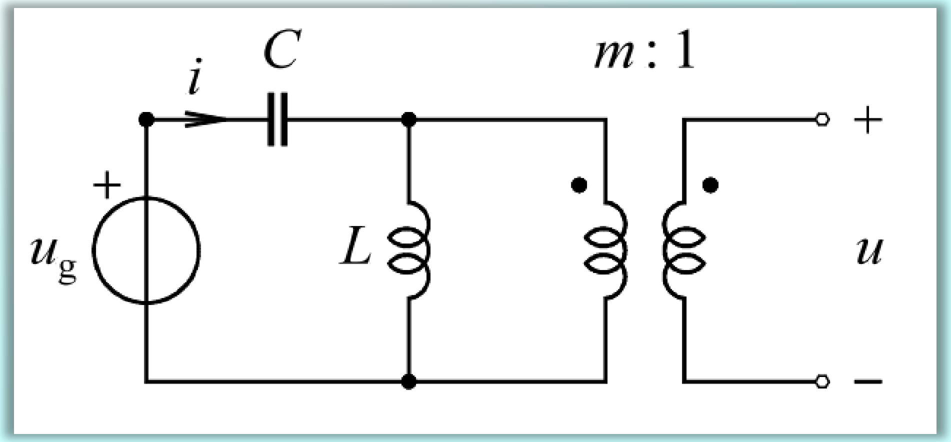
(5) Одредити једначину одзива за напон u електричног кола са слике. Вредности елемената и параметри побуде су познати,

$$u_{\mathrm{g}}(t) = U_{\mathrm{m}} \sin(\frac{1}{\sqrt{CL}} t)\,\mathrm{h}(t)\,.$$

Нема сакупљене енергије.

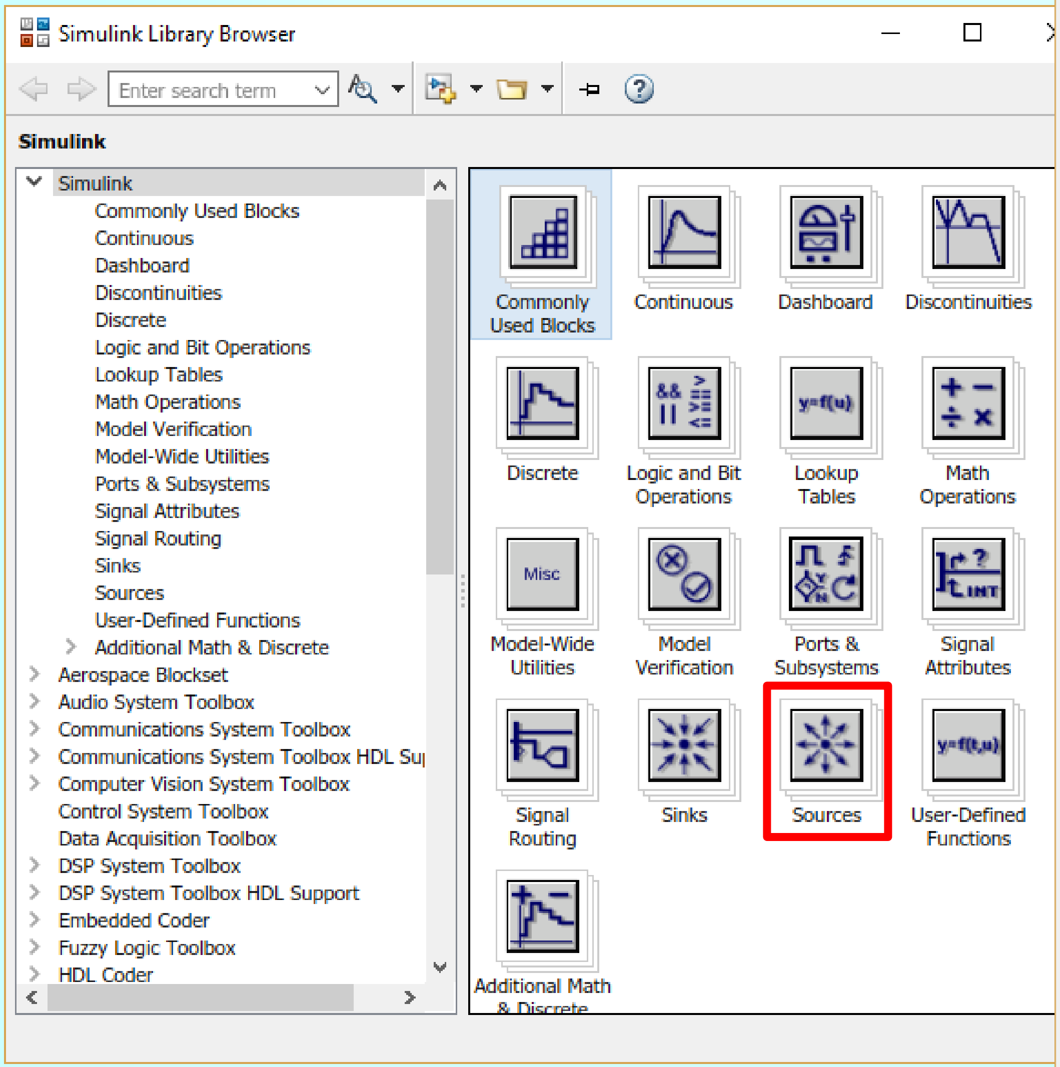
(5) Одредити струју извора, i, и нацртати график струје извора.

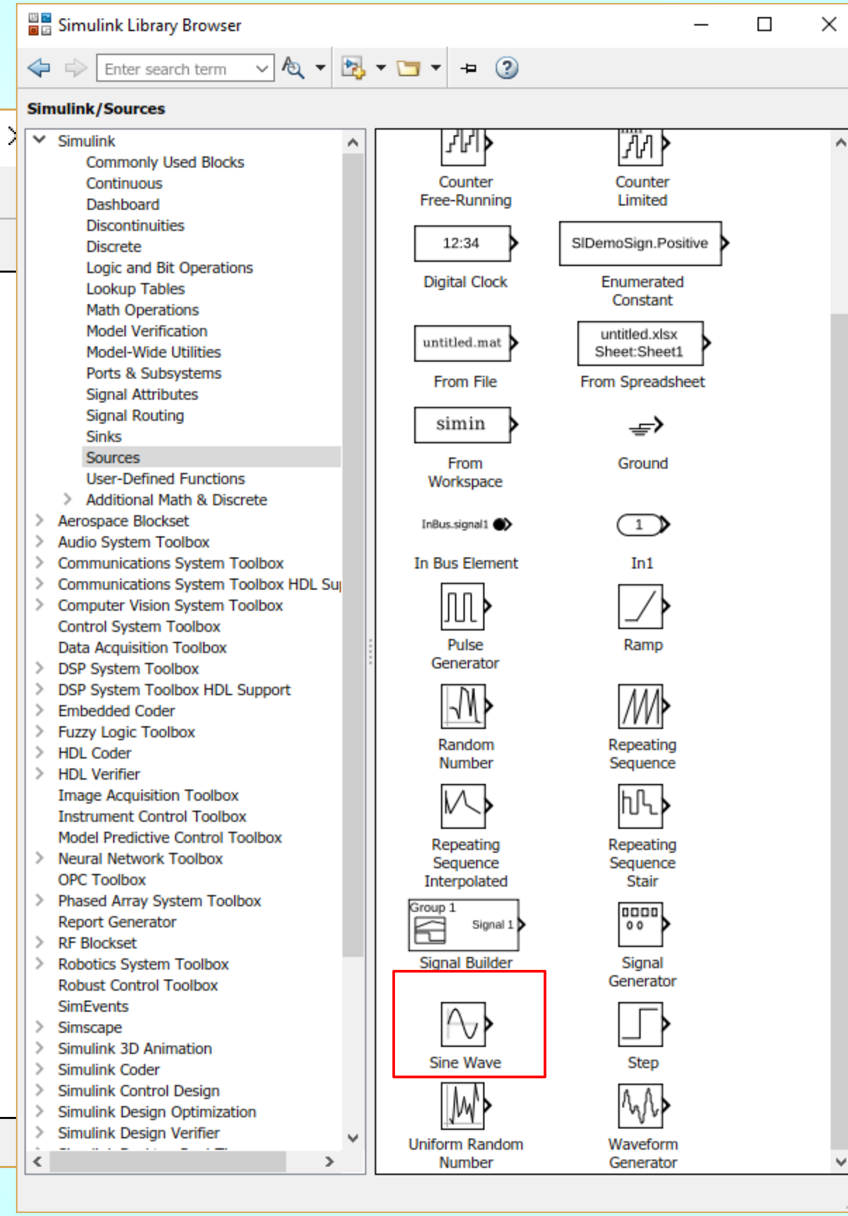
(5) Како гласе једначине стања кола? Написати их у матричном облику.

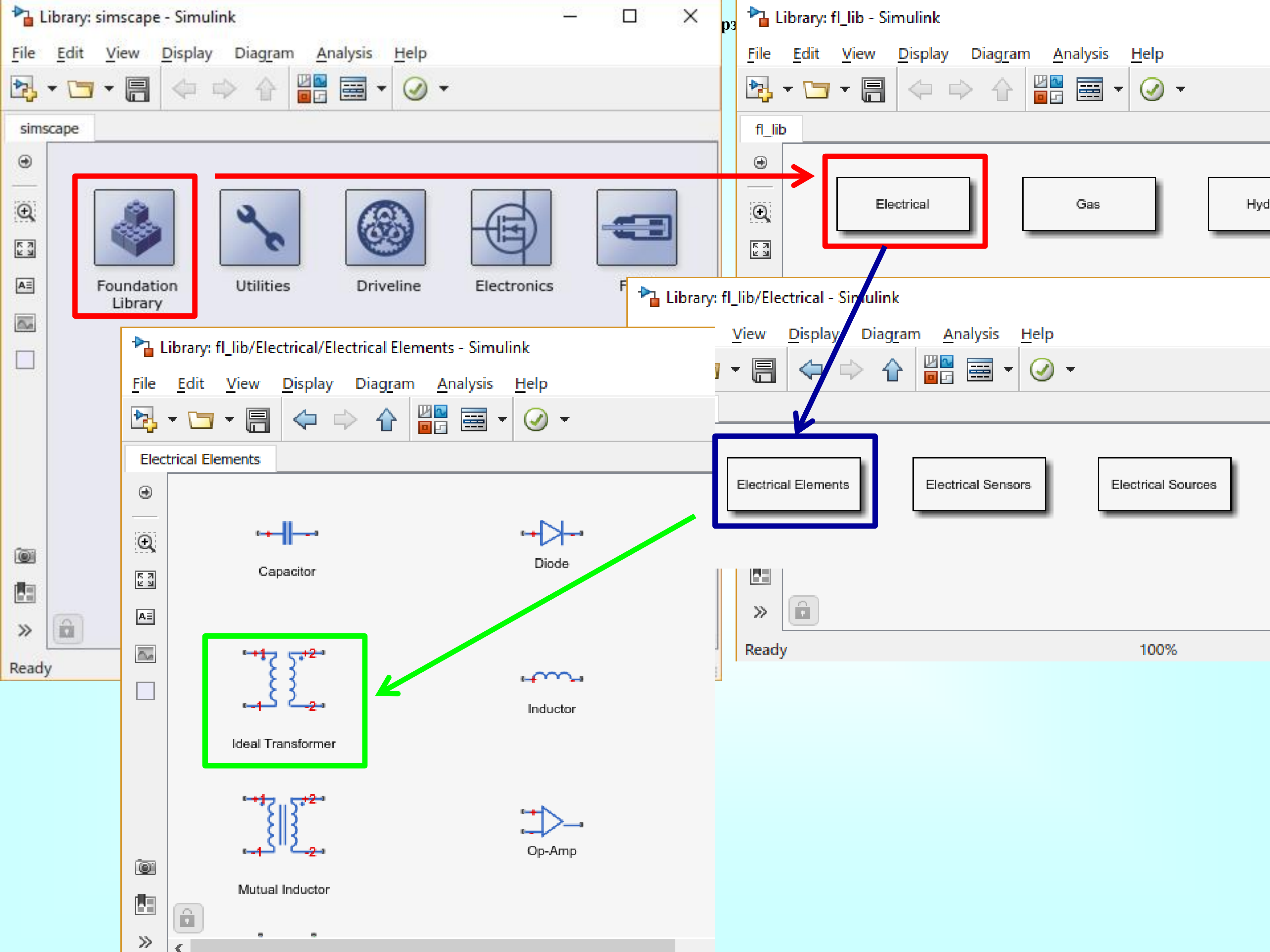


MATLAB: Simulink

Sources

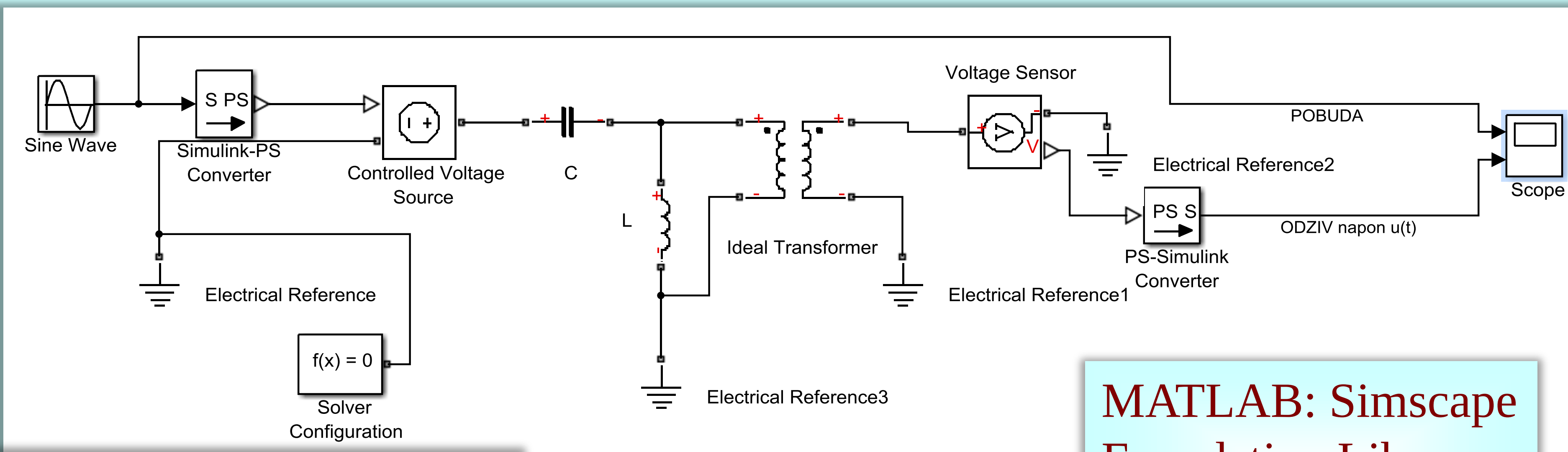


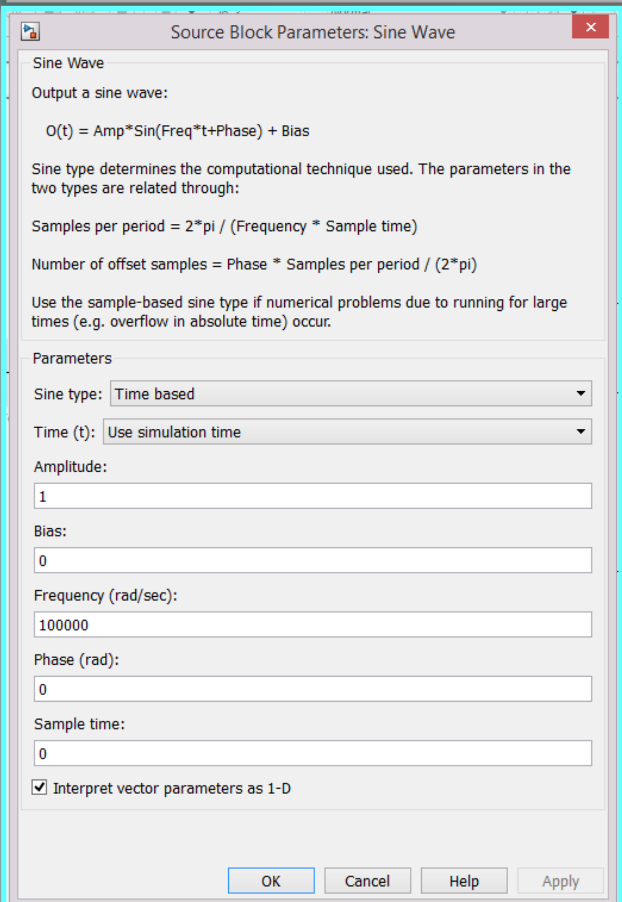


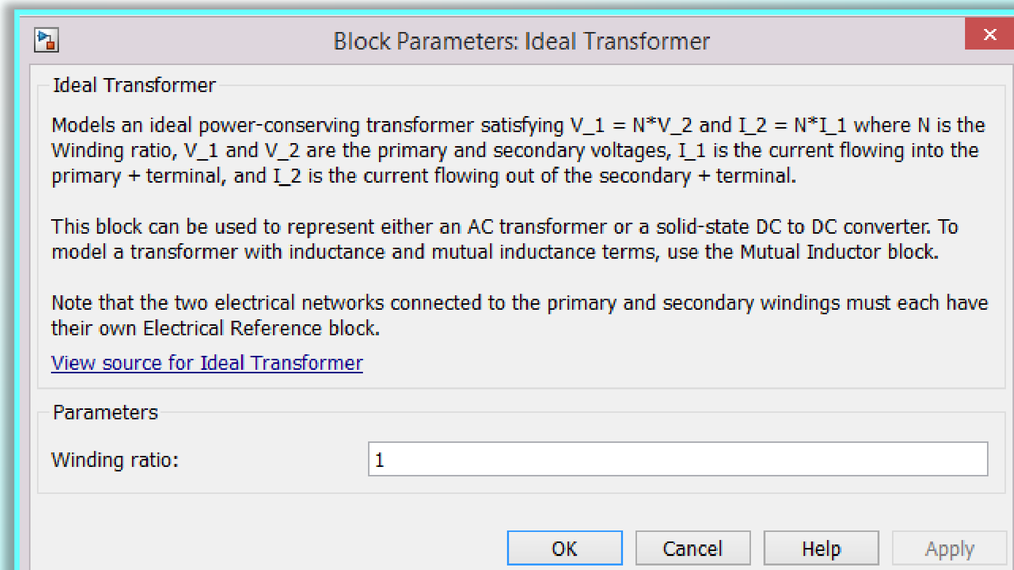


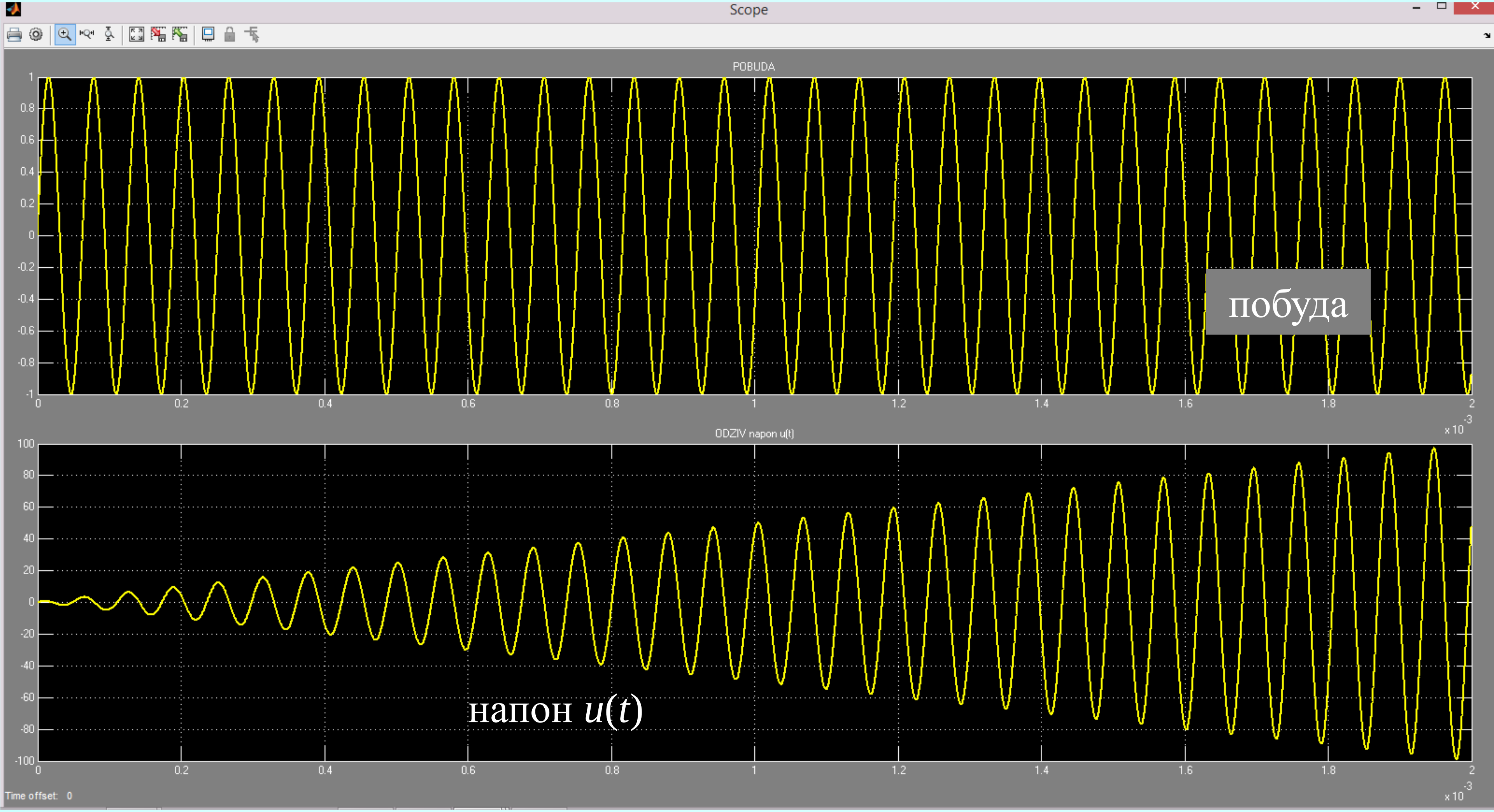
Library: simscape - Simulink
File Edit View Display Diagram Analysis Help
simscape
Foundation Library
Utilities
Driveline
Electronics
Library: fl_lib - Simulink
fl_lib
Electrical
Gas
Library: fl_lib/Electrical - Simulink
Electrical Elements
Electrical Sensors
Electrical Sources
Ready
100%
Library: fl_lib/Electrical/Electrical Elements - Simulink
Capacitor
Diode
Ideal Transformer
Inductor
Mutual Inductor
Op-Amp
Ready

MATLAB: Simscape Foundation Library Utilities Simulink








Scope
POBUDA
побуда
ODZIV napon u(t)
напон $u(t)$
Time offset: 0

MATLAB: MuPAD

```
jednacine:={ug = uC + uL,
uL = L * Di,
i = C*DuC,
uL=m*u}
```

$$\{\mathrm{uL} = m\,u,\ \mathrm{ug} = \mathrm{uC} + \mathrm{uL},\ i = C\,\mathrm{DuC},\ \mathrm{uL} = \mathrm{Di}\,L\}$$

```
jednacineIzvoda:=groebner::eliminate(jednacine,{uL, u})
```

$$[\mathrm{uC} - \mathrm{ug} + \mathrm{Di}\,L,\ i - C\,\mathrm{DuC}]$$

```
assume(C>0 and L>0 and m>0 and Um>0 and t>=0)
jednacineStanja := solve(jednacineIzvoda, {Di, DuC})
```

$$\left\{\left[\mathrm{Di} = -\frac{\mathrm{uC} - \mathrm{ug}}{L},\ \mathrm{DuC} = \frac{i}{C}\right]\right\}$$

```
PromenljiveStanjaIzvodi:={uC=uC(t), i=i(t), Di=D(i)(t), DuC=D(uC)(t)}
```

$$\{i = i(t),\ \mathrm{uC} = \mathrm{uC}(t),\ \mathrm{Di} = i'(t),\ \mathrm{DuC} = \mathrm{uC}'(t)\}$$

```
pobuda := {ug=Um*sin(t/sqrt(L*C))*heaviside(t)}
```

$$\left\{\mathrm{ug} = \mathrm{Um}\,\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)\mathrm{heaviside}(t)\right\}$$

JS:=jednacineStanja | PromenljiveStanjaIzvodi | pobuda

$$\left\{\left[i'(t) = -\frac{\mathrm{uC}(t) - \mathrm{Um}\sin\left(\frac{t}{\sqrt{C}\sqrt{L}}\right)\mathrm{heaviside}(t)}{L},\ \mathrm{uC}'(t) = \frac{i(t)}{C}\right]\right\}$$

op(op(JS))

$$i'(t) = -\frac{\mathrm{uC}(t) - \mathrm{Um}\sin\left(\frac{t}{\sqrt{C}\sqrt{L}}\right)\mathrm{heaviside}(t)}{L},\ \mathrm{uC}'(t) = \frac{i(t)}{C}$$

JSpocetveVrednosti:={op(op(JS)), uC(0)=0, i(0)=0}

$$\left\{\mathrm{uC}'(t) = \frac{i(t)}{C},\ i(0) = 0,\ \mathrm{uC}(0) = 0,\ i'(t) = -\frac{\mathrm{uC}(t) - \mathrm{Um}\sin\left(\frac{t}{\sqrt{C}\sqrt{L}}\right)\mathrm{heaviside}(t)}{L}\right\}$$

odziv := ode::solve(JSpocetveVrednosti, {uC(t), i(t)})

$$\left\{\left[\mathrm{uC}(t) = \frac{\mathrm{Um}\,\sigma_1}{4} + \frac{\mathrm{Um}\,\sigma_1\,\mathrm{sign}(t)}{4} - \frac{\mathrm{Um}\,t\,\sigma_2}{4\sqrt{C}\sqrt{L}} - \frac{\mathrm{Um}\,t\,\sigma_2\,\mathrm{sign}(t)}{4\sqrt{C}\sqrt{L}},\ i(t) = \frac{\mathrm{Um}\,t\,\sigma_1}{4L} + \frac{\mathrm{Um}\,t\,\sigma_1\,\mathrm{sign}(t)}{4L}\right]\right\}$$

where

$$\sigma_1 = \sin\left(\frac{t}{\sqrt{C}\sqrt{L}}\right)$$

$$\sigma_2 = \cos\left(\frac{t}{\sqrt{C}\sqrt{L}}\right)$$

MATLAB: MuPAD

MATLAB: MuPAD

assume(t>0):(uC(t) | odziv)

$$\frac{\mathrm{Um}\,\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2}-\frac{\mathrm{Um}\,t\,\cos\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2\,\sqrt{C}\,\sqrt{L}}$$

assume(t>0):(i(t) | odziv)

$$\frac{\mathrm{Um}\,t\,\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2\,L}$$

iL(t):=i(t) | odziv

$$\frac{\mathrm{Um}\,t\,\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2\,L}$$

uL(t):=Simplify(L*(diff(iL(t),t)))

$$\frac{\mathrm{Um}\,\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2}+\frac{\mathrm{Um}\,t\,\cos\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2\,\sqrt{C}\,\sqrt{L}}$$

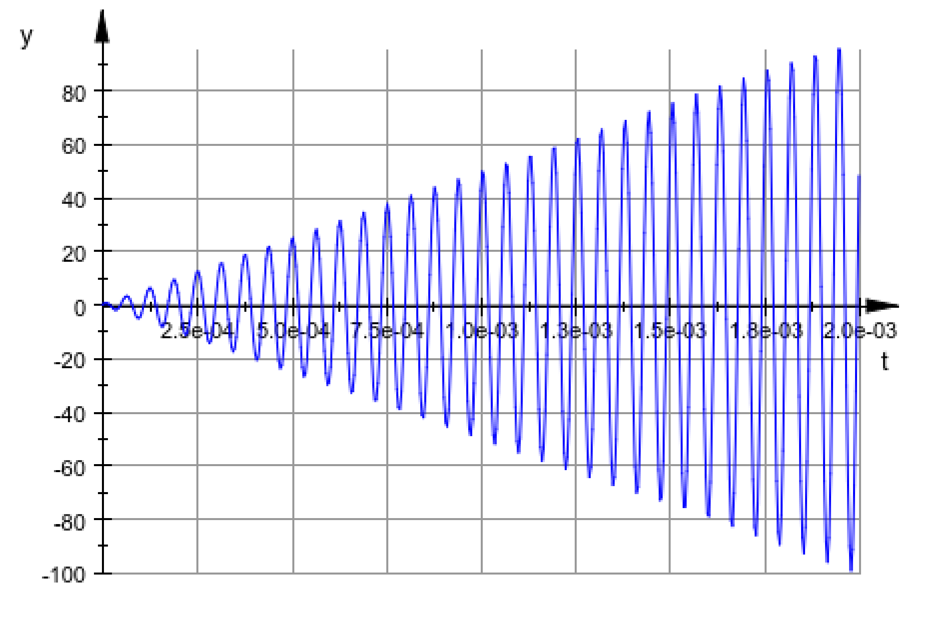
u(t):=m*uL(t)

$$m\left(\frac{\mathrm{Um}\,\sin\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2}+\frac{\mathrm{Um}\,t\,\cos\left(\frac{t}{\sqrt{C}\,\sqrt{L}}\right)}{2\,\sqrt{C}\,\sqrt{L}}\right)$$

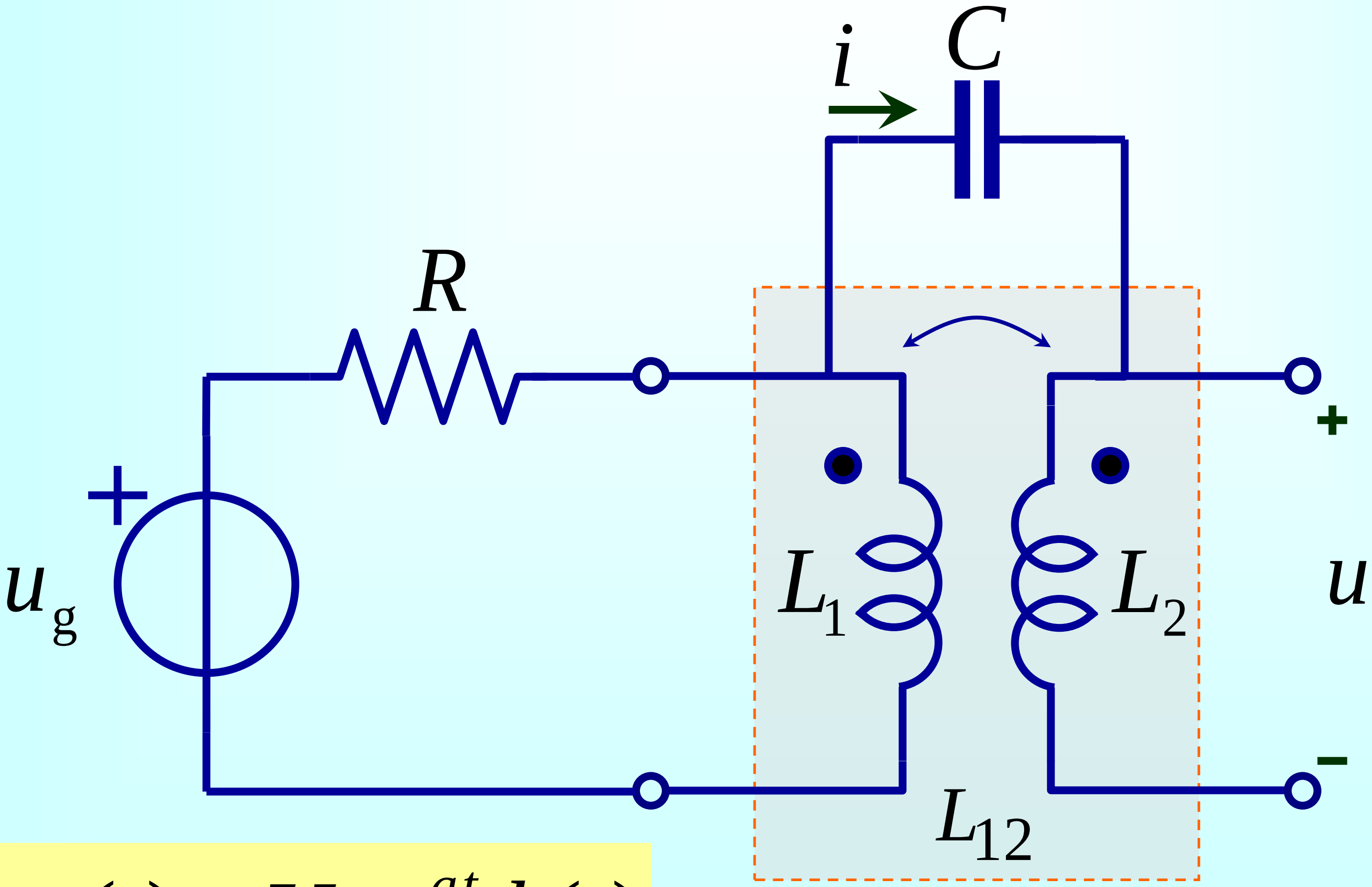
zamena:={L=0.1e-3, C=1e-6, Um=1, m=1}

$$\{L=0.0001,\ \mathrm{Um}=1,\ m=1,\ C=0.000001\}$$

plot(u(t)|zamena, t=0..2e-3, GridVisible = TRUE)



Одзив на експоненцијалну побуду



$L = 1\text{mH}$

$C = 1\mu\text{F}$

$R = 1\text{k}\Omega$

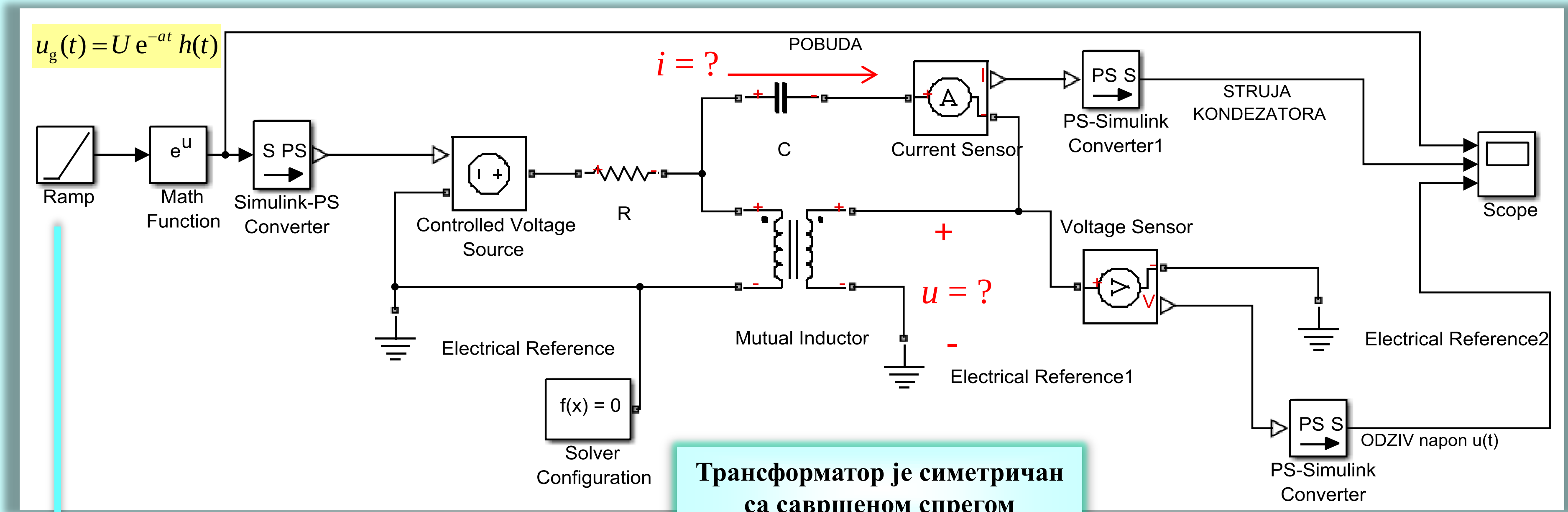
$$u_g(t) = U\,\mathrm{e}^{-at}\,h(t)$$

$U = 1\text{V},\ a = 100$

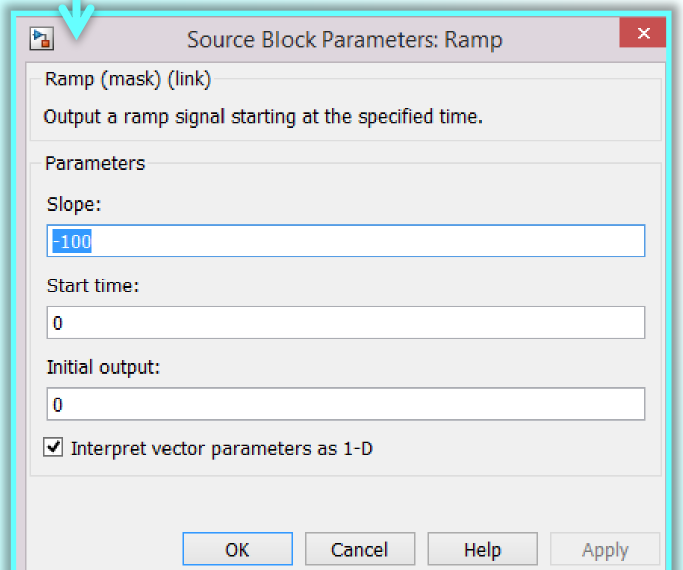
Трансформатор је симетричан са савршеном спрегом

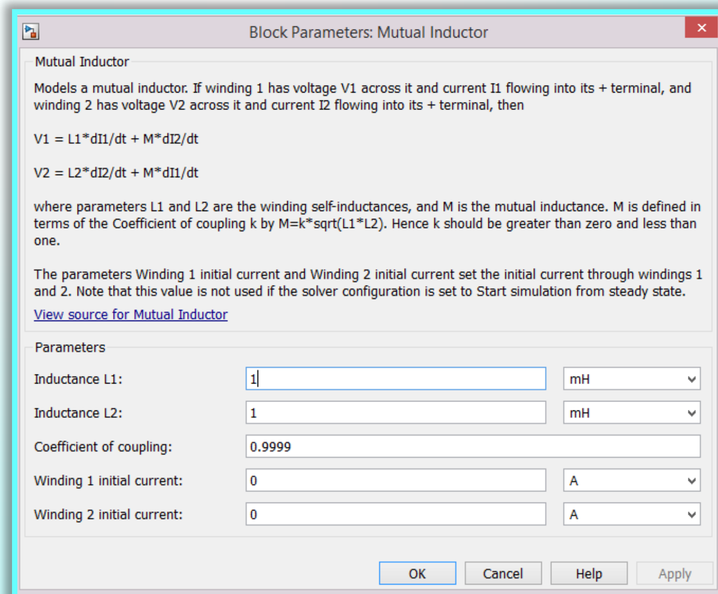
Задатак - експоненцијална побуда

- Параметри кола са слике су познати. Линеарни индуктивни трансформатор је симетричан са савршеном спрегом. Нема сакупљене енергије.
- Одредити струју кондензатора $i(t)$ и нацртати њен график.
- Одредити напон секундара $u(t)$ и нацртати његов график.



Трансформатор је симетричан са савршеном спрегом





$$L = 1\,\mathrm{mH}$$

$$C = 1\,\mu\mathrm{F}$$

$$R = 1\,\mathrm{k}\Omega$$

MATLAB: Simscape
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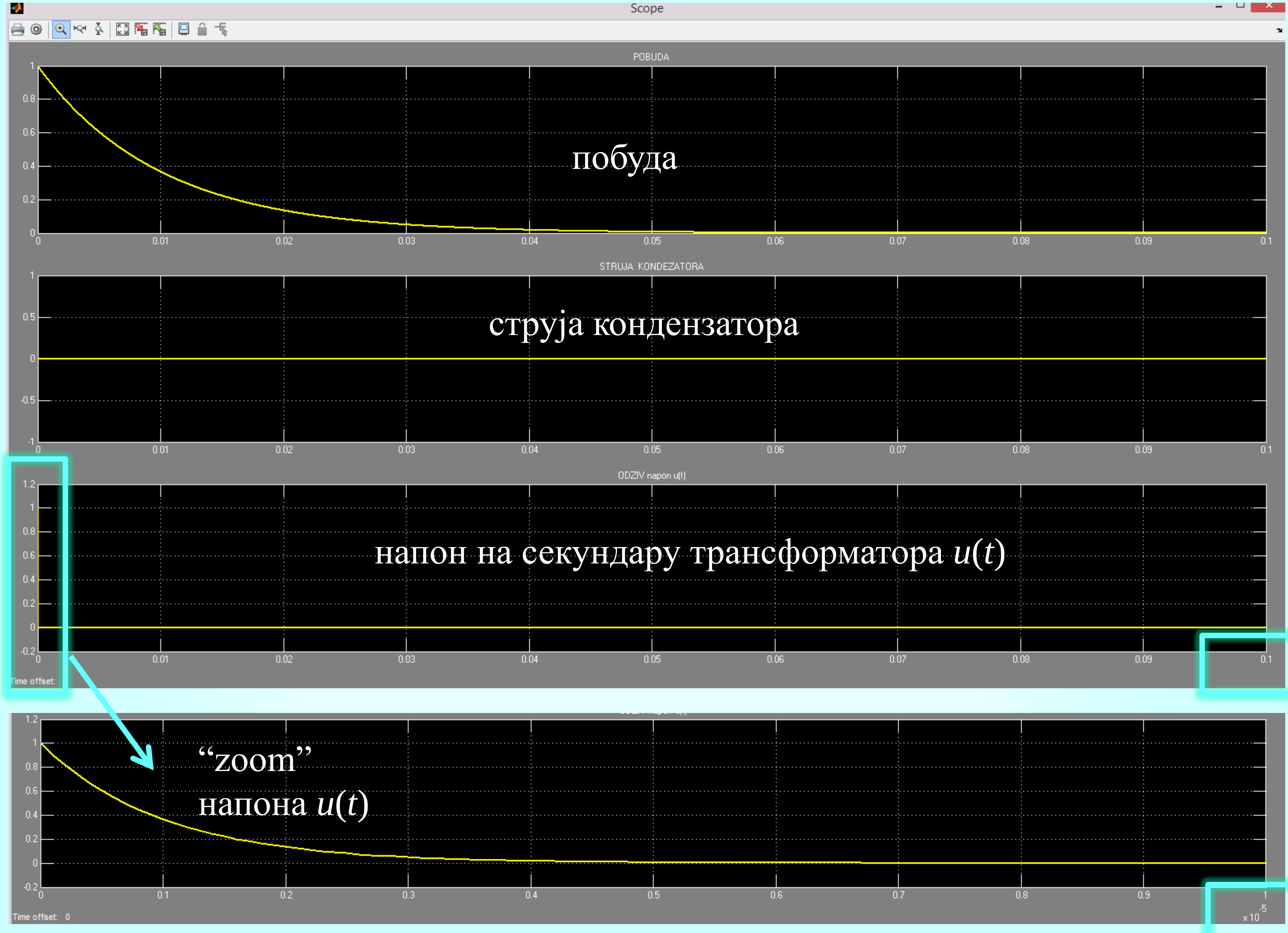
$$U = 1\mathrm{V},\ a = 100$$


Scope
POBUDA
побуда
STRUJA KONDEZATORA
струја кондензатора
ODZIV napon u(t)
напон на секундару трансформатора $u(t)$
Time offset:
“zoom”
напона $u(t)$
Time offset: 0
$x\,10^{-5}$

MATLAB: MuPAD

```
assume( R>0 and C>0 and L1>0 and L2>0 and L12>0 and L>0 and U>0 and a>0)
zamene := {L1=L, L2=L, L12=L}
```

$\{L1 = L,\ L2 = L,\ L12 = L\}$

```
pobuda:={ug=U*exp(-a*t)*heaviside(t)}
```

$\{ug = U\,e^{-a\,t}\,\mathrm{heaviside}(t)\}$

```
jednacine:={ug = uR + uL1,
uR = R*iR,
uL1 = L1*DiL1 + L12*DiL2,
u = L2*DiL2 + L12*DiL1,
uC = uL1 - u,
i = C*DuC,
iR = i + iL1,
i = iL2}
```

$\{u = \mathrm{DiL2}\ \mathrm{L2} + \mathrm{DiL1}\ \mathrm{L12},\ \mathrm{uL1} = \mathrm{DiL1}\ \mathrm{L1} + \mathrm{DiL2}\ \mathrm{L12},\ \mathrm{ug} = \mathrm{uL1} + \mathrm{uR},\ i = \mathrm{iL2},\ \mathrm{uC} = \mathrm{uL1} - u,\ i = C\ \mathrm{DuC},\ \mathrm{uR} = R\ \mathrm{iR},\ \mathrm{iR} = i + \mathrm{iL1}\}$

```
promenljive := {uL1, u, uR, iR, i}
```

$\{i,\ \mathrm{iR},\ u,\ \mathrm{uL1},\ \mathrm{uR}\}$

```
jednacineIzvoda:=groebner::eliminate(jednacine, promenljive)
```

$[R\ \mathrm{iL1} - \mathrm{ug} + R\ \mathrm{iL2} + \mathrm{DiL1}\ \mathrm{L1} + \mathrm{DiL2}\ \mathrm{L12},\ C\ \mathrm{DuC} - \mathrm{iL2},\ \mathrm{uC} - \mathrm{DiL1}\ \mathrm{L1} + \mathrm{DiL2}\ \mathrm{L2} + \mathrm{DiL1}\ \mathrm{L12} - \mathrm{DiL2}\ \mathrm{L12}]$

```
JednacineIzvodaZamene:=jednacineIzvoda | zamene
```

$[R\ \mathrm{iL1} - \mathrm{ug} + R\ \mathrm{iL2} + \mathrm{DiL1}\ L + \mathrm{DiL2}\ L,\ C\ \mathrm{DuC} - \mathrm{iL2},\ \mathrm{uC}]$

```
PromenljiveStanjaIzvodi:={uC=uC(t), DuC=D(uC)(t), iL1=iL1(t), DiL1=D(iL1)(t), iL2=iL2(t), DiL2=D(iL2)(t)}
```

$\{\mathrm{iL1} = \mathrm{iL1}(t),\ \mathrm{iL2} = \mathrm{iL2}(t),\ \mathrm{uC} = \mathrm{uC}(t),\ \mathrm{DiL1} = \mathrm{iL1}'(t),\ \mathrm{DiL2} = \mathrm{iL2}'(t),\ \mathrm{DuC} = \mathrm{uC}'(t)\}$

JednacinePromenljiveStanja:=JednacineIzvodaZamene | PromenljiveStanjaIzvodi | {pobuda}

$$\left[L\,\mathrm{iL1}'(t) + R\,\mathrm{iL1}(t) + L\,\mathrm{iL2}'(t) + R\,\mathrm{iL2}(t) - U\,\mathrm{e}^{-a\,t}\,\mathrm{heaviside}(t),\ C\,\mathrm{uC}'(t) - \mathrm{iL2}(t),\ \mathrm{uC}(t)\right]$$

promenljiveStanja:=[iL1(t), iL2(t), uC(t)]

$$[\mathrm{iL1}(t),\ \mathrm{iL2}(t),\ \mathrm{uC}(t)]$$

MATLAB: MuPAD

```
[noveJednacine, novePromenljive,
 reseneJednacine,
 konstantnePromenljive,
 zamenjenePromenljive,
 drugeJednacine
] := daetools::reduceRedundancies(JednacinePromenljiveStanja, promenljiveStanja)
```

$$\left[\left[\mathrm{e}^{-a\,t}\left(L\,\mathrm{e}^{a\,t}\,\mathrm{iL1}'(t) + R\,\mathrm{e}^{a\,t}\,\mathrm{iL1}(t) - U\,\mathrm{heaviside}(t)\right)\right],\ [\mathrm{iL1}(t)],\ [\mathrm{uC}(t),\ -\mathrm{iL2}(t)],\ [\mathrm{uC}(t) = 0,\ \mathrm{iL2}(t) = 0],\ [\,],\ [\,]\right]$$

noveJednacine

$$\left[\mathrm{e}^{-a\,t}\left(L\,\mathrm{e}^{a\,t}\,\mathrm{iL1}'(t) + R\,\mathrm{e}^{a\,t}\,\mathrm{iL1}(t) - U\,\mathrm{heaviside}(t)\right)\right]$$

novePromenljive

$$[\mathrm{iL1}(t)]$$

reseneJednacine

$$[\mathrm{uC}(t),\ -\mathrm{iL2}(t)]$$

konstantnePromenljive

$$[\mathrm{uC}(t) = 0,\ \mathrm{iL2}(t) = 0]$$

zamenjenePromenljive

$$[\,]$$

drugeJednacine

$$[\,]$$

noveJednacinePocetveVrednosti:={(op(noveJednacine)), iL1(0)=0}

$$\left\{ e^{-a\,t} \left(L\, e^{a\,t}\, \mathrm{iL1}'(t) + R\, e^{a\,t}\, \mathrm{iL1}(t) - U\, \mathrm{heaviside}(t) \right),\ \mathrm{iL1}(0) = 0 \right\}$$

odziv := ode::solve(noveJednacinePocetveVrednosti, {iL1(t)})

$$\left\{ -e^{-\frac{R\,t}{L}} \left(\frac{U\, \mathrm{heaviside}(t)}{R - L\,a} - \frac{U\, e^{\frac{R\,t}{L}}\, e^{-a\,t}\, \mathrm{heaviside}(t)}{R - L\,a} \right) \right\}$$

iL1:=t--> op(odziv)

$$t \to -e^{-\frac{R\,t}{L}} \left(\frac{U\, \mathrm{heaviside}(t)}{R - L\,a} - \frac{U\, e^{\frac{R\,t}{L}}\, e^{-a\,t}\, \mathrm{heaviside}(t)}{R - L\,a} \right)$$

u(t):=L12*diff(iL1(t),t) | zamene

$$-L \left(\sigma_2 \left(\frac{U\, \delta(t)}{R-L\,a} - \frac{U\, \sigma_1\, e^{-a\,t}\, \delta(t)}{R-L\,a} + \frac{U\, a\, \sigma_1\, e^{-a\,t}\, \mathrm{heaviside}(t)}{R-L\,a} - \frac{R\,U\, \sigma_1\, e^{-a\,t}\, \mathrm{heaviside}(t)}{L\,(R-L\,a)} \right) - \frac{R\, \sigma_2 \left(\frac{U\, \mathrm{heaviside}(t)}{R-L\,a} - \frac{U\, \sigma_1\, e^{-a\,t}\, \mathrm{heaviside}(t)}{R-L\,a} \right)}{L} \right)$$

where

$$\sigma_1 = e^{\frac{R\,t}{L}}$$

$$\sigma_2 = e^{-\frac{R\,t}{L}}$$

MATLAB: MuPAD

MATLAB: MuPAD

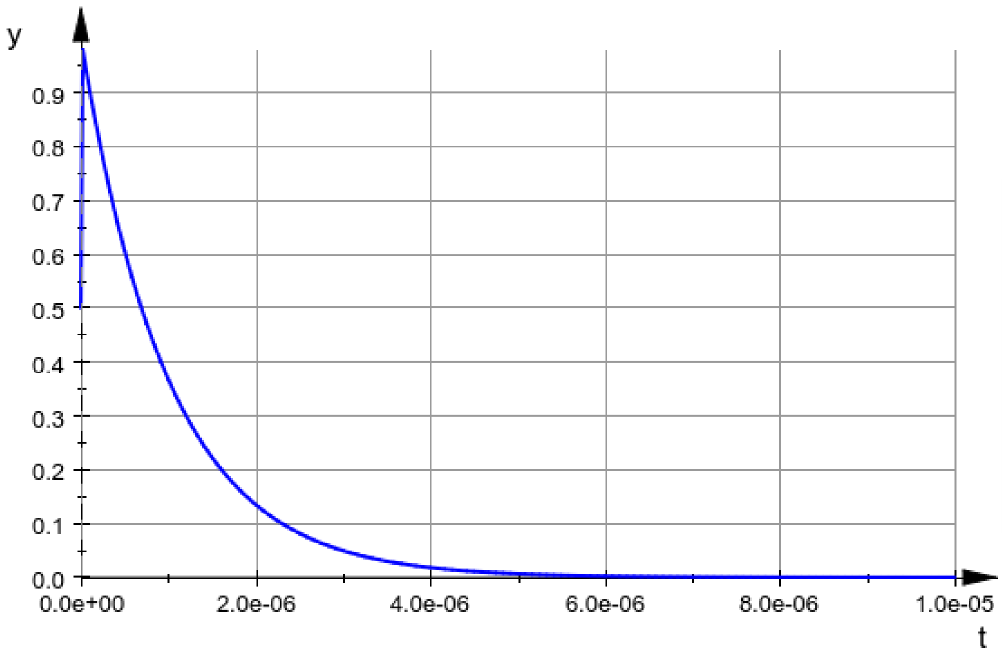
u:=t--> Simplify(u(t))

$$t \to \frac{U\,\mathrm{heaviside}(t)\,\left(R\,\mathrm{e}^{-\frac{R\,t}{L}} - L\,a\,\mathrm{e}^{-a\,t}\right)}{R - L\,a}$$

vrednosti:={R=1e3, C=1e-6, L=1e-3, U=1, a=100 }

$$\{R = 1000.0,\ U = 1,\ a = 100,\ C = 0.000001,\ L = 0.001\}$$

plot(u(t) | vrednosti, t=0..1e-5, GridVisible = TRUE)



limit(u(t) | vrednosti, t = 0, Right)

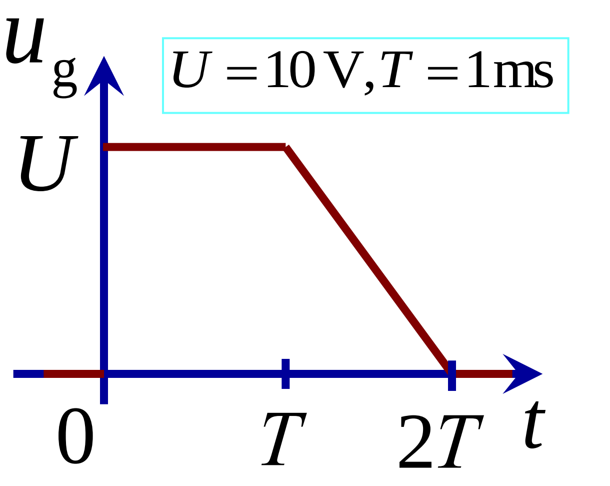
1.0

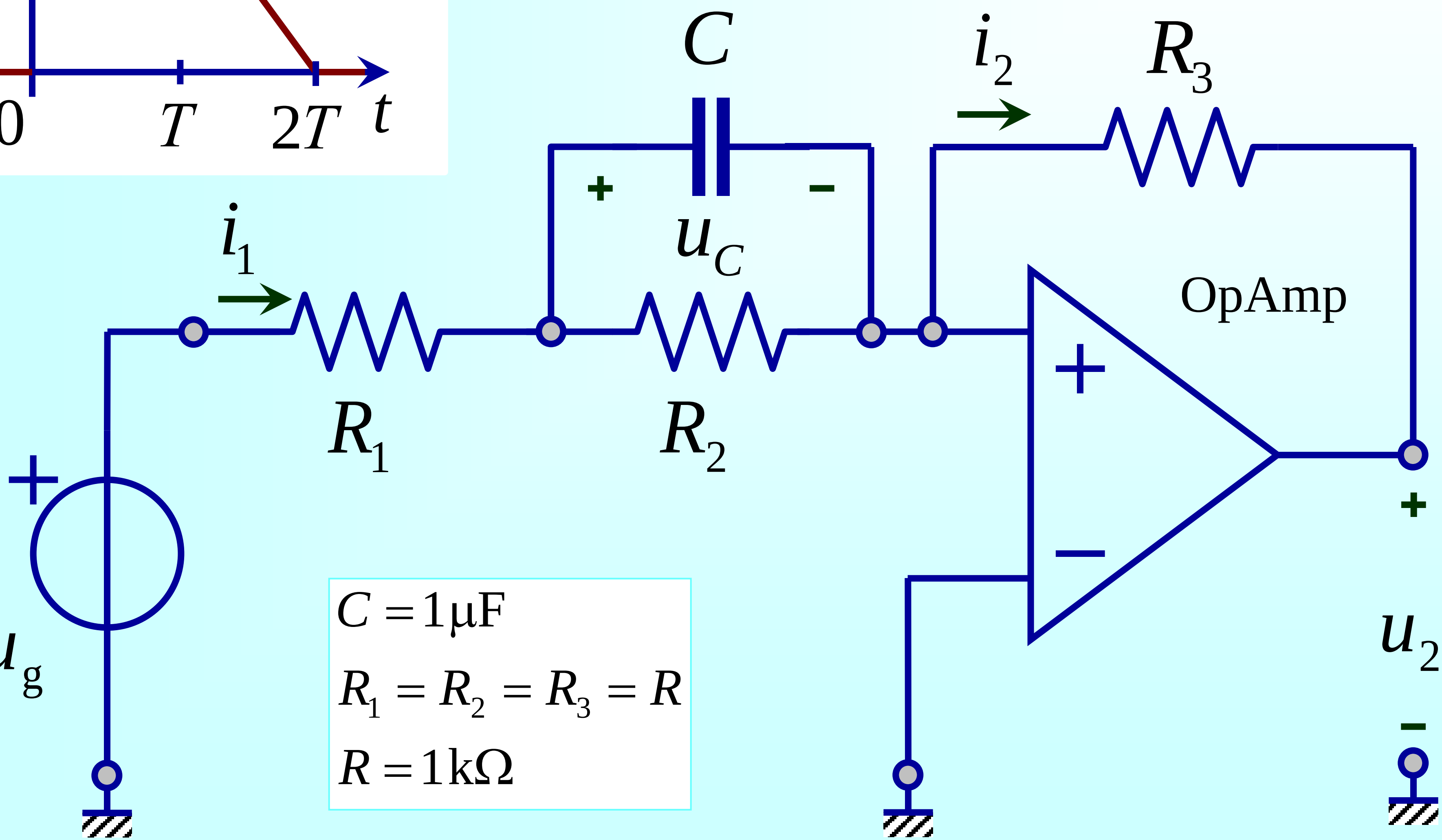
limit(u(t) | vrednosti, t = 0, Left)

0

Гранична вредност за напон $u(t)$ кад $t \to 0$

Одзив на сложену побуду





$C = 1\,\mu\mathrm{F}$

$R_1 = R_2 = R_3 = R$

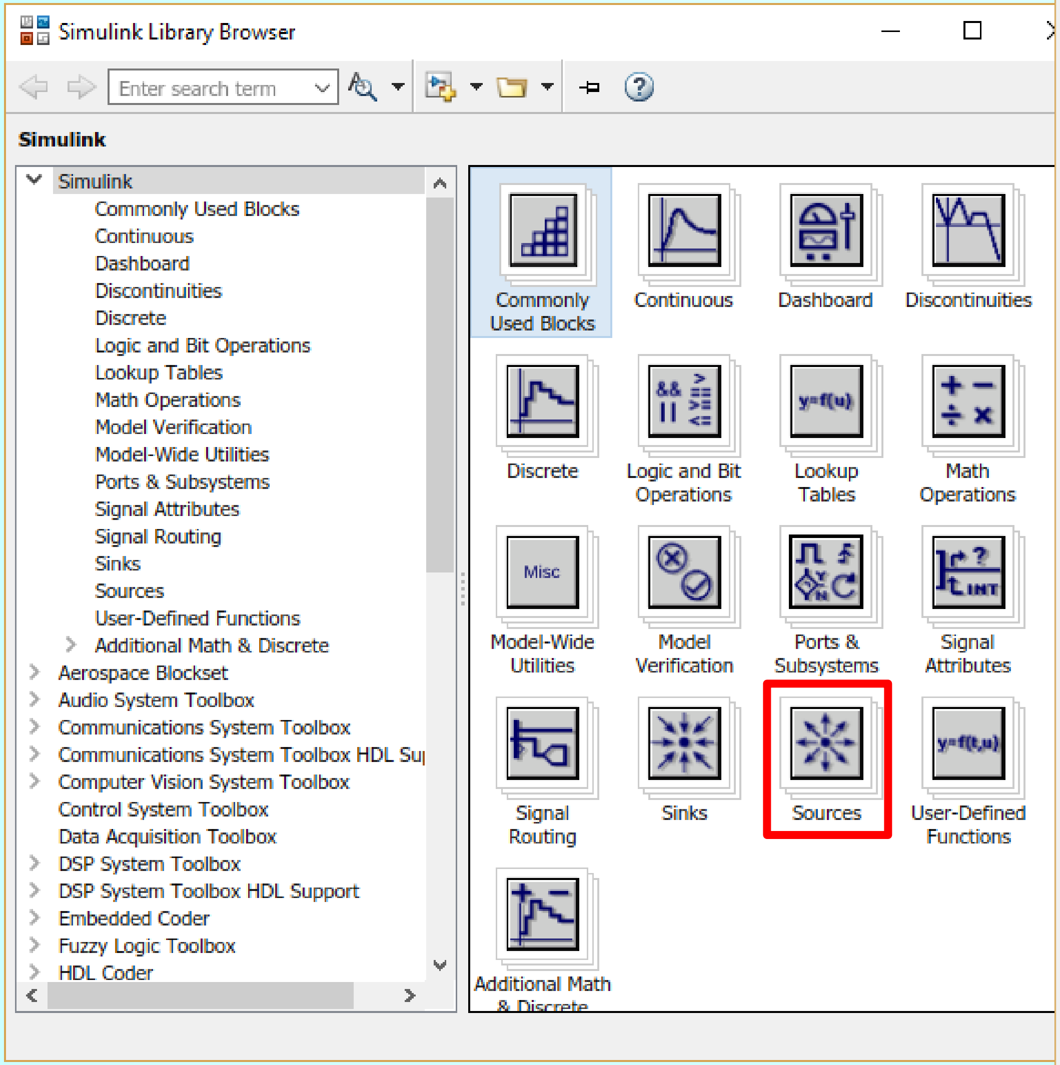
$R = 1\,\mathrm{k}\Omega$

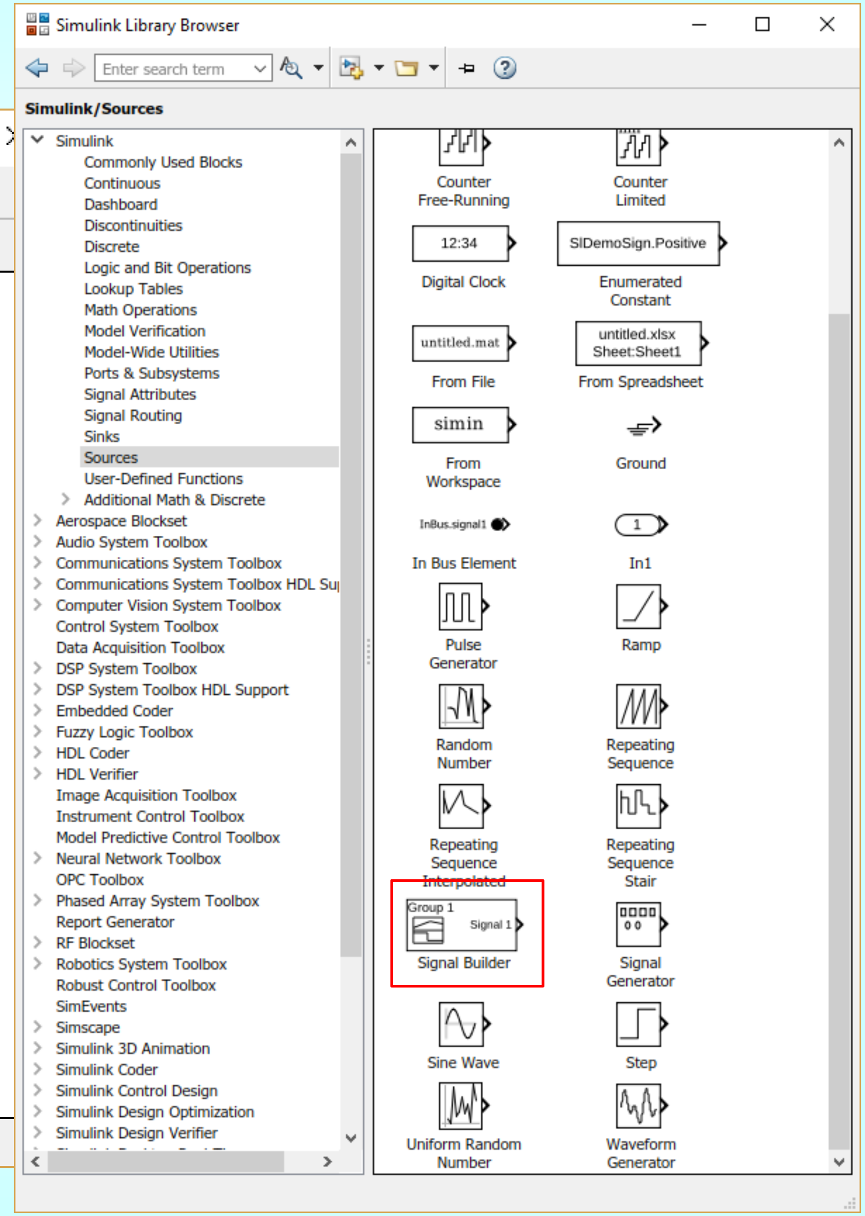
Задатак - сложена побуда

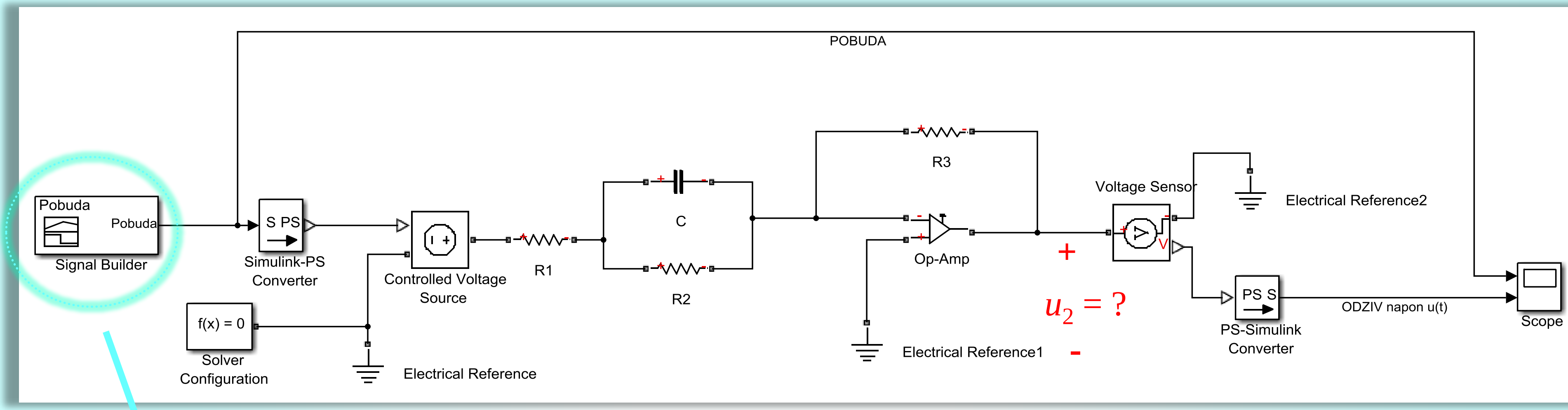
- Параметри кола са слике су познати: $R_1 = R_2 = R_3 = R$. Нема сакупљене енергије.
- Применом конволуционог интеграла одредити напон кондензатора $u_C(t)$, а затим и излазни напон $u_2(t)$ и нацртати његов график.
- Напон генератора се мења са временом као на дијаграму, при чему је $T = RC$.

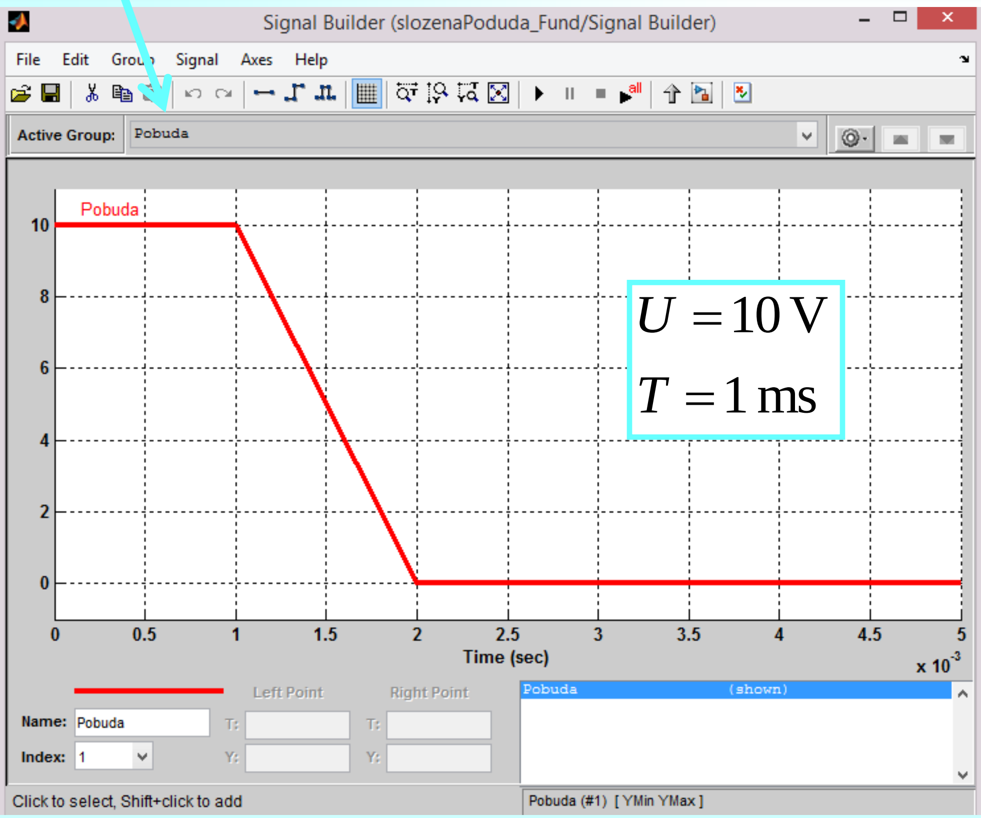
MATLAB: Simulink

Sources







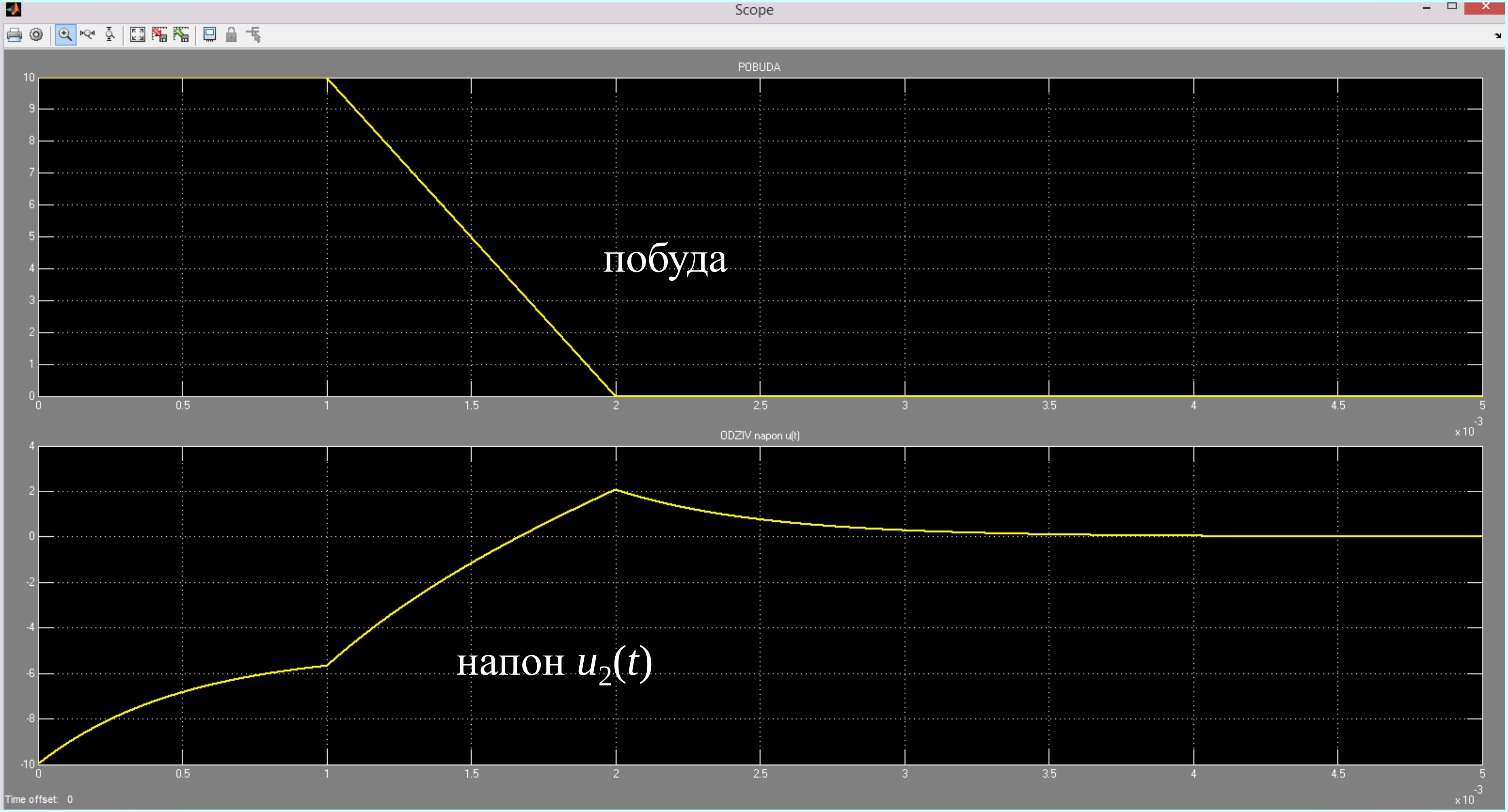


$$u_2 = ?$$

$$R_1 = R_2 = R_3 = 1\,\mathrm{k\Omega}$$

$$C_1 = 1\,\mu\mathrm{F}$$

MATLAB: Simscape
Foundation Library
Utilities
Simulink


Scope
POBUDA
побуда
ODZIV napon u(t)
напон $u_2(t)$
Time offset: 0
x 10^-3

MATLAB: MuPAD

```
assume( R1>0 and R2>0 and R3>0 and R>0 and C>0 and U>0 and T>0)
jednacine:={ug = uR1 + uC,
uR1=R1*i1,
iC=C*DuC,
uR2=R2*iR2,
uR2=uC,
i1 =  iR2+iC,
i2=i1,
u=-R3*i2}
```

$$\{\mathrm{ug} = \mathrm{uC} + \mathrm{uR1},\ \mathrm{i2} = \mathrm{i1},\ \mathrm{uR2} = \mathrm{uC},\ \mathrm{iC} = C\,\mathrm{DuC},\ u = -\mathrm{R3}\ \mathrm{i2},\ \mathrm{uR1} = \mathrm{R1}\ \mathrm{i1},\ \mathrm{uR2} = \mathrm{R2}\ \mathrm{iR2},\ \mathrm{i1} = \mathrm{iC} + \mathrm{iR2}\}$$

```
jednacineIzvoda:=groebner::eliminate(jednacine,{ i1, iC, uR1, iR2, uR2, i2, u})
```

$$[\mathrm{R1}\ \mathrm{uC} + \mathrm{R2}\ \mathrm{uC} - \mathrm{R2}\ \mathrm{ug} + C\ \mathrm{DuC}\ \mathrm{R1}\ \mathrm{R2}]$$

```
jednacineStanje := solve(jednacineIzvoda, {DuC})
```

$$\left\{\left[\mathrm{DuC} = -\frac{\mathrm{R1}\ \mathrm{uC} + \mathrm{R2}\ \mathrm{uC} - \mathrm{R2}\ \mathrm{ug}}{C\ \mathrm{R1}\ \mathrm{R2}}\right]\right\}$$

```
PromenljiveStanjaIzvodi:={uC=uC(t), DuC=D(uC)(t)}
```

$$\{\mathrm{uC} = \mathrm{uC}(t),\ \mathrm{DuC} = \mathrm{uC}'(t)\}$$

```
zamene := {R1=R, R2=R, R3=R, T=R*C}
```

$$\{\mathrm{R1} = R,\ \mathrm{R2} = R,\ \mathrm{R3} = R,\ T = C\ R\}$$

MATLAB: MuPAD

```
pobuda := {ug=heaviside(t)}
```

$\{ug = \mathrm{heaviside}(t)\}$

```
JS:=jednacineStanje | PromenljiveStanjaIzvodi | {pobuda, zamene}
```

$$\left\{\left[uC'(t) = \frac{R\,\mathrm{heaviside}(t) - 2\,R\,uC(t)}{C\,R^2}\right]\right\}$$

```
JSpocetveVrednosti:={op(op(JS)), uC(0)=0}
```

$$\left\{uC'(t) = \frac{R\,\mathrm{heaviside}(t) - 2\,R\,uC(t)}{C\,R^2},\ uC(0) = 0\right\}$$

```
odziv := ode::solve(JSpocetveVrednosti, {uC(t)})
```

$$\left\{\frac{e^{-\frac{2\,t}{C\,R}}\,\mathrm{heaviside}(t)\left(e^{\frac{2\,t}{C\,R}} - 1\right)}{2}\right\}$$

Индициона функција за $u_C(t)$

```
f_uC(t):=(op(odziv))
```

$$\frac{e^{-\frac{2\,t}{C\,R}}\,\mathrm{heaviside}(t)\left(e^{\frac{2\,t}{C\,R}} - 1\right)}{2}$$

Импулсни одзив за $u_C(t)$

```
g_uC:=t --> simplify(diff(f_uC(t),t))
```

$$t \to \frac{e^{-\frac{2\,t}{C\,R}}\,\mathrm{heaviside}(t)}{C\,R}$$

```
ug_1:=t -> U
```

$t \to U$

```
ug_2:= t-> -U*(t-2*T)/T
```

$$t \to -\frac{U\,(t - 2\,T)}{T}$$

```
ug_3:= t-> 0
```

$t \to 0$

$0 < t \leq T$

```
assume(0< t <= T and T > 0): uC_1(t):=int(ug_1(Tau) * g_uC(t-Tau), Tau=0..t)
```

$$-\frac{U\left(e^{-\frac{2t}{CR}}-1\right)}{2}$$

Конволуциони интеграл за напон $u_C(t)$ за $0 < t \leq T$

```
i1_1(t):=(1/R)*(ug_1(t)-uC_1(t))|zamene:
i2_1(t):=i1_1(t):
u2_1(t):=-R*i2_1(t)
```

$$-U-\frac{U\left(e^{-\frac{2t}{CR}}-1\right)}{2}$$

```
Simplify(u2_1(t))
```

$$-\frac{U\left(e^{-\frac{2t}{CR}}+1\right)}{2}$$

напон $u_2(t)$ за $0 < t \leq T$

MATLAB: MuPAD

```
assume(2*T >= t > T and T>0):
uC_2(t):=int(ug_1(Tau) * g_uC(t-Tau), Tau=0..T) + int(ug_2(Tau) * g_uC(t-Tau), Tau=T..t)
```

$$\frac{U\,e^{-\frac{2t}{CR}}\left(e^{\frac{2T}{CR}}-1\right)}{2} - U\left(e^{\frac{2(T-t)}{CR}}-1\right) - \frac{U\left(\frac{CR(2t-CR)}{4} - \frac{CR\,e^{\frac{2T-2t}{CR}}(2T-CR)}{4}\right)}{CRT}$$

Конволуциони интеграл за напон $u_C(t)$ за $T < t \leq 2T$

```
Simplify(uC_2(t)|zamene)
```

$$\frac{5U}{4} - \frac{U\,e^{-\frac{2t}{CR}}}{2} - \frac{U\,e^2\,e^{-\frac{2t}{CR}}}{4} - \frac{U\,t}{2CR}$$

$T < t \leq 2T$

```
i1_2(t):=(1/R)*(ug_2(t) - uC_2(t))|zamene:
i2_2(t):=i1_2(t):
u2_2(t):=-R*i2_2(t)
```

$$\frac{U\,e^{-\frac{2t}{CR}}\left(e^2-1\right)}{2} - U\left(e^{-\frac{2(t-CR)}{CR}}-1\right) - \frac{U\left(\frac{CR(2t-CR)}{4} - \frac{C^2R^2\,e^{-\frac{2t-2CR}{CR}}}{4}\right)}{C^2R^2} + \frac{U(t-2CR)}{CR}$$

```
Simplify(u2_2(t))
```

$$\frac{U\,t}{2CR} - \frac{U\,e^{-\frac{2t}{CR}}}{2} - \frac{U\,e^2\,e^{-\frac{2t}{CR}}}{4} - \frac{3U}{4}$$

напон $u_2(t)$ за $T < t \leq 2T$

MATLAB: MuPAD

```
assume(t > 2*T and T > 0):
uC_3(t):=int(ug_1(Tau) * g_uC(t-Tau), Tau=0..T)+int(ug_2(Tau) * g_uC(t-Tau), Tau=T..2*T)
```

$$\frac{U\, e^{-\frac{2t}{CR}} \left(e^{\frac{2T}{CR}} - 1\right)}{2} - \frac{U\, e^{\frac{2T}{CR}}\, e^{-\frac{2t}{CR}} \left(2\,T + C\,R - C\,R\, e^{\frac{2T}{CR}}\right)}{4\,T}$$

Конволуциони интеграл за напон $u_C(t)$ за $t > 2T$

```
Simplify(uC_3(t)|zamene)
```

$$\frac{U\, e^{-\frac{2t}{CR}} \left(e^{2} + 1\right) \left(e^{2} - 2\right)}{4}$$

$t > 2T$

```
i1_3(t):=(1/R)*(ug_3(t)-uC_3(t)) | zamene:
i2_3(t):=i1_3(t):
u2_3(t):=-R*i2_3(t)
```

$$\frac{U\, e^{-\frac{2t}{CR}} \left(e^{2} - 1\right)}{2} - \frac{U\, e^{2}\, e^{-\frac{2t}{CR}} \left(3\,C\,R - C\,R\, e^{2}\right)}{4\,C\,R}$$

```
Simplify(u2_3(t))
```

$$\frac{U\, e^{-\frac{2t}{CR}} \left(e^{2} + 1\right) \left(e^{2} - 2\right)}{4}$$

напон $u_2(t)$ за $t > 2T$

MATLAB: MuPAD

vrednosti:={R=1e3, C=1e-6, U=10, T=1e-3 }

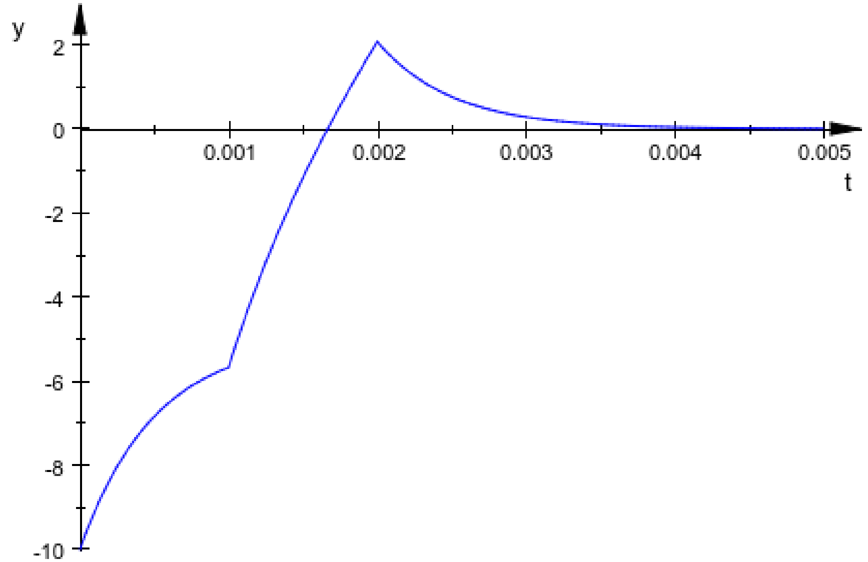
MATLAB: MuPAD

$$\{R = 1000.0,\ U = 10,\ C = 0.000001,\ T = 0.001\}$$

u2(t) := piecewise([0 < t <= T | vrednosti, u2_1(t)],
[T < t <= 2*T | vrednosti, u2_2(t)], [t > 2*T | vrednosti, u2_3(t)]) | vrednosti

$$\begin{cases} 5\,e^{-2000.0\,t}\left(e^2-1\right)+2500.0\,e^2\,e^{-2000.0\,t}\left(0.001\,e^2-0.003\right) & \text{if } 0.002 < t \\ -5\,e^{-2000.0\,t}-5 & \text{if } t \le 0.001 \\ 5000.0\,t-7.5\,e^{2.0-2000.0\,t}+5\,e^{-2000.0\,t}\left(e^2-1\right)-7.5 & \text{if } t \in (0.001,\ 0.002] \end{cases}$$

plot(u2(t) | zamene, tx=0..5e-3)



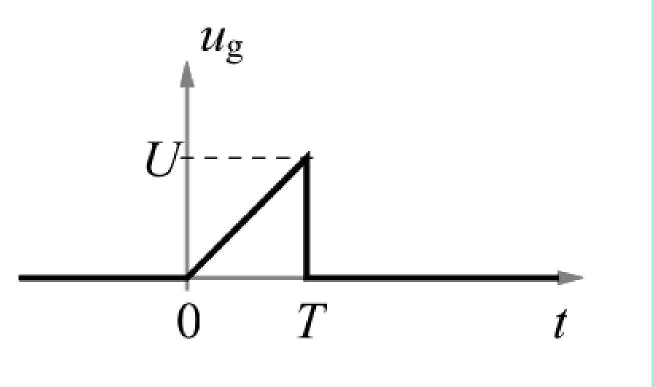
Одзив на троугаони импулс

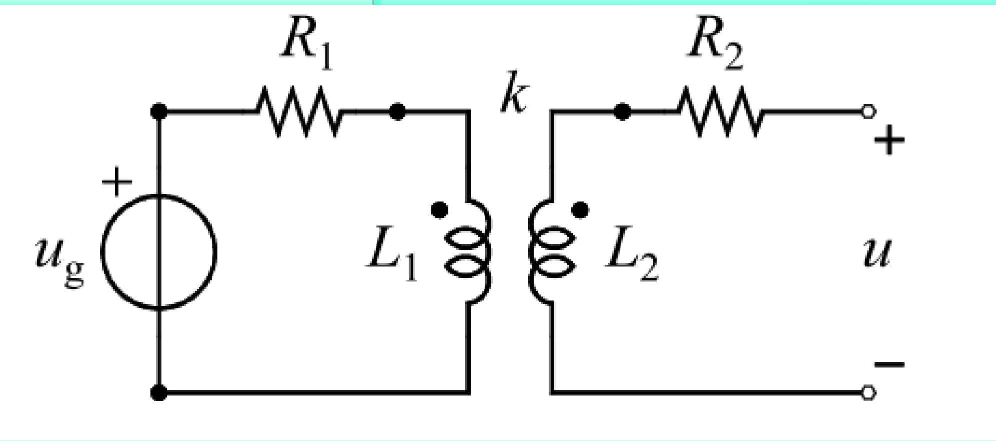
Параметри електричног кола са слике су познати. Побуда је дата на слици, $T = L/R$, $R_1 = R$, $R_2 = 2R$, $L_1 = L$, $L_2 = 4L$, $k = 1/2$.

(5) Одредити индициону функцију за напон отвореног секундара (одскочни одзив).

(5) Одредити напон отвореног секундара и

(5) нацртати његов график. Обележити осе графика, координатни почетак, пресеке и додире графика са осама, и тачке екстремума.





```
assume( R1>0 and L1>0 and L2>0 and L12>0 and L>0 and U>0 and T>0 and t>=0)
zamene := {L1=L, L2=4*L, L12=(1/2)*sqrt(L*4*L), R1=R, R2=2*R, T=L/R}
```

$$\left\{\mathrm{R1} = R,\ T = \frac{L}{R},\ \mathrm{L2} = 4\ L,\ \mathrm{R2} = 2\ R,\ \mathrm{L1} = L,\ \mathrm{L12} = L\right\}$$

```
jednacine:={ug = uR1 + u1,
u1 = L1 * Di1  ,
u2 = L12 * Di1,
uR1=R1*i1}
```

$$\{\mathrm{ug} = \mathrm{u1} + \mathrm{uR1},\ \mathrm{u1} = \mathrm{Di1}\ \mathrm{L1},\ \mathrm{u2} = \mathrm{Di1}\ \mathrm{L12},\ \mathrm{uR1} = \mathrm{R1}\ \mathrm{i1}\}$$

```
Jednacine:=jednacine | zamene
```

$$\{\mathrm{ug} = \mathrm{u1} + \mathrm{uR1},\ \mathrm{u1} = \mathrm{Di1}\ L,\ \mathrm{u2} = \mathrm{Di1}\ L,\ \mathrm{uR1} = R\ \mathrm{i1}\}$$

```
jednacineIzvoda:=groebner::eliminate(Jednacine,{u1,  u2, uR1})
```

$$[R\ \mathrm{i1} - \mathrm{ug} + \mathrm{Di1}\ L]$$

```
jednacineStanja := solve(jednacineIzvoda, {Di1})
```

$$\left\{\left[\mathrm{Di1} = \frac{\mathrm{ug} - R\ \mathrm{i1}}{L}\right]\right\}$$

```
PromenljiveStanjaIzvodi:={i1=i1(t), Di1=D(i1)(t)}
```

$$\{\mathrm{i1} = \mathrm{i1}(t),\ \mathrm{Di1} = \mathrm{i1}'(t)\}$$

MATLAB: MuPAD

pobuda := {ug=heaviside(t)}

$$\{ug = \mathrm{heaviside}(t)\}$$

JS:=jednacineStanja | PromenljiveStanjaIzvodi | pobuda

$$\left\{\left[\mathrm{i1}'(t) = \frac{\mathrm{heaviside}(t) - R\,\mathrm{i1}(t)}{L}\right]\right\}$$

op(op(JS))

$$\mathrm{i1}'(t) = \frac{\mathrm{heaviside}(t) - R\,\mathrm{i1}(t)}{L}$$

JSpocetveVrednosti:={op(op(JS)), i1(0)=0}

$$\left\{\mathrm{i1}'(t) = \frac{\mathrm{heaviside}(t) - R\,\mathrm{i1}(t)}{L}, \boxed{\mathrm{i1}(0) = 0}\right\}$$

odziv := ode::solve(JSpocetveVrednosti, {i1(t)})

$$\left\{\frac{e^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)\left(e^{\frac{R\,t}{L}} - 1\right)}{R}\right\}$$

f_i1(t):=(op(odziv))

Индициона функција за струју $i_1(t)$

$$\frac{e^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)\left(e^{\frac{R\,t}{L}} - 1\right)}{R}$$

MATLAB: MuPAD

```
g_i1:=t --> simplify((diff(f_i1(t),t)))
```

Импулсни одзив за струју $i_1(t)$

$$t \to \frac{e^{-\frac{R\,t}{L}}\,\mathrm{heaviside}(t)}{L}$$

```
assume(0<t<=T):i1_1(t):=int((U*Tau/T)*g_i1(t-Tau), Tau=0..t)
```

$$\frac{U\left(R\,t - L + L\,e^{-\frac{R\,t}{L}}\right)}{R^2\,T}$$

Конволуциони интеграл за струју $i_1(t)$ за $0 < t \leq T$

```
u2_1(t):=simplify(L12*diff(i1_1(t),t)|zamene)
```

$$\boxed{U - U\,e^{-\frac{R\,t}{L}}}$$

напон $u_2(t)$ за $0 < t \leq T$

```
assume(t>T): i1_2(t):=int((U*Tau/T)*g_i1(t-Tau), Tau=0..T)
```

$$\frac{L\,U\,e^{-\frac{R\,t}{L}}}{R^2\,T} - \frac{U\,e^{\frac{R\,T - R\,t}{L}}\,(L - R\,T)}{R^2\,T}$$

Конволуциони интеграл за струју $i_1(t)$ за $t > T$

```
u2_2(t):=simplify(L12*diff(i1_2(t),t)|zamene)
```

$$\boxed{-U\,e^{-\frac{R\,t}{L}}}$$

напон $u_2(t)$ за $t > T$

MATLAB: MuPAD

vrednosti:={R=1000, L=1e-3, T=1e-6, U=10}

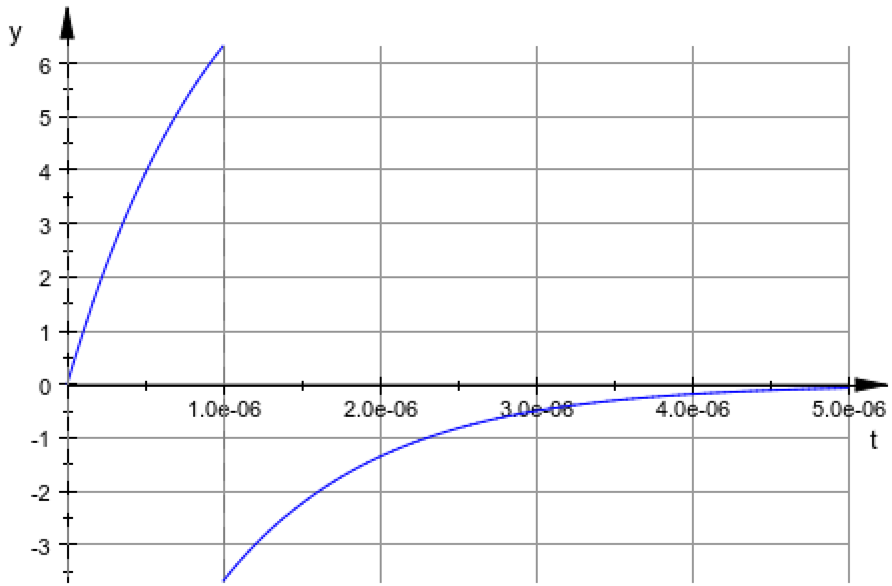
$$\{R = 1000,\ U = 10,\ L = 0.001,\ T = 0.000001\}$$

u2(t) := piecewise([0 <t <= T|vrednosti, u2_1(t)], [t > T|vrednosti, u2_2(t)])|vrednosti

$$\begin{cases} -10\,e^{-1000000.0\,t} & \text{if } 0.000001 < t \\ 10 - 10\,e^{-1000000.0\,t} & \text{if } t \in (0,\ 0.000001] \end{cases}$$

напон $u_2(t)$ за $t > 0$

plot(u2(t),t=0..5e-6, GridVisible = TRUE)



limit(u2(t), t = 1e-6, Left)

$$6.321205588$$

limit(u2(t), t = 1e-6, Right)

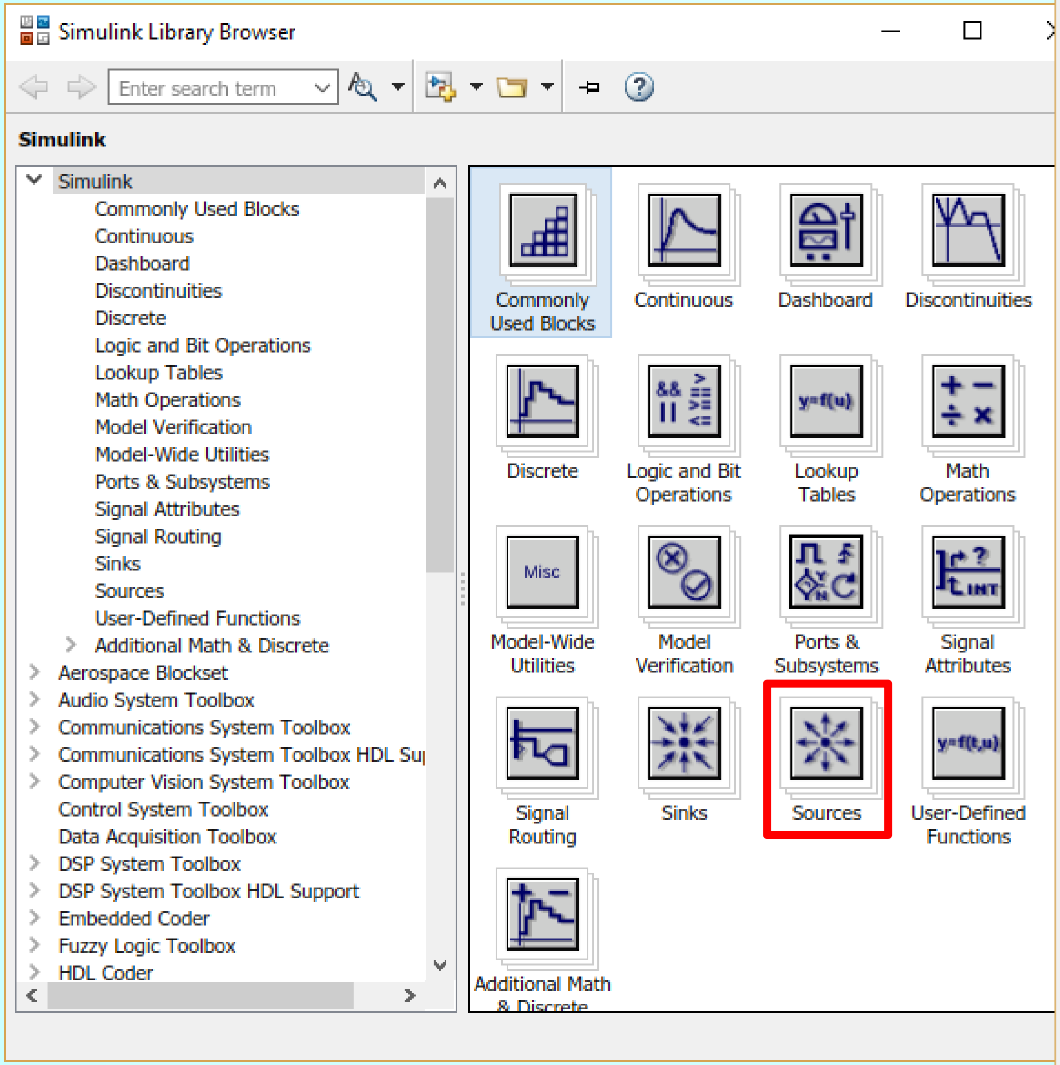
$$-3.678794412$$

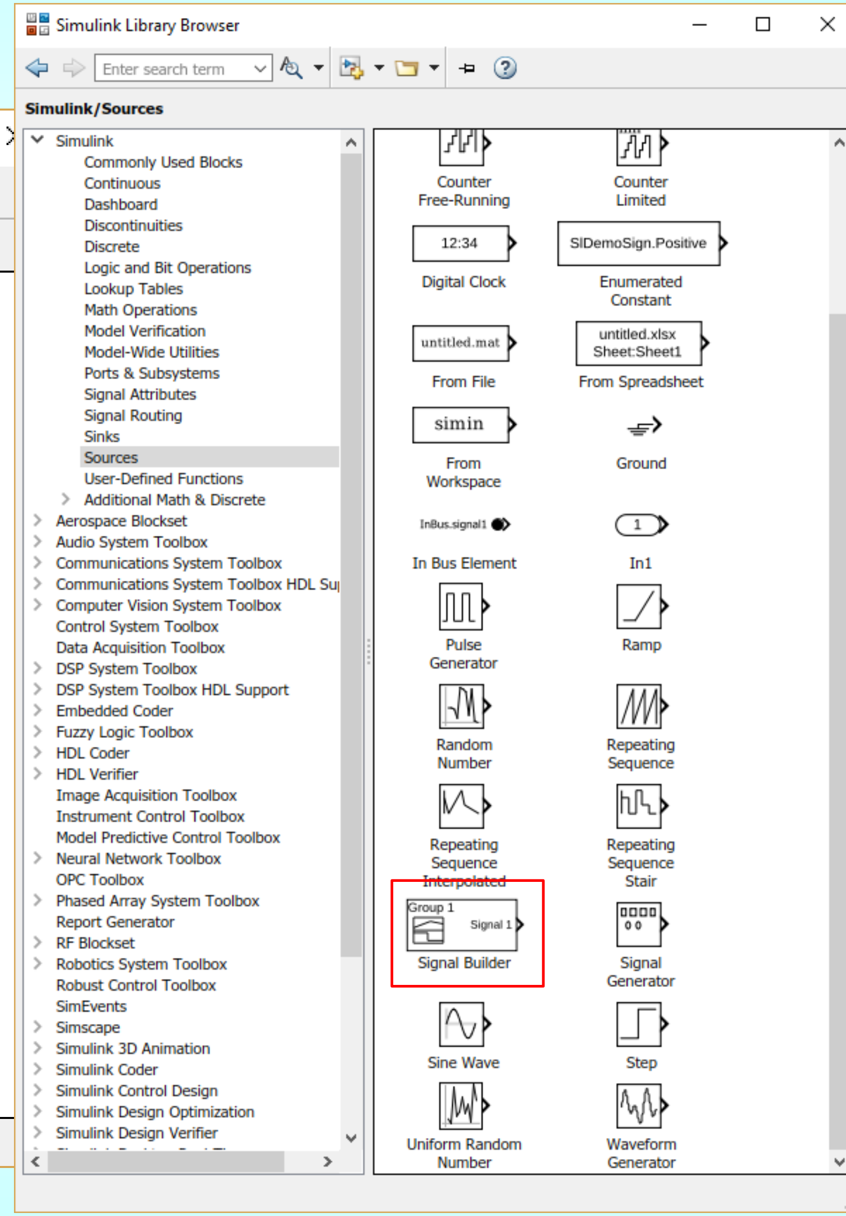
Гранична вредност за напон $u_2(t)$ кад $t \to T = 1\ \mu s$

MATLAB: MuPAD

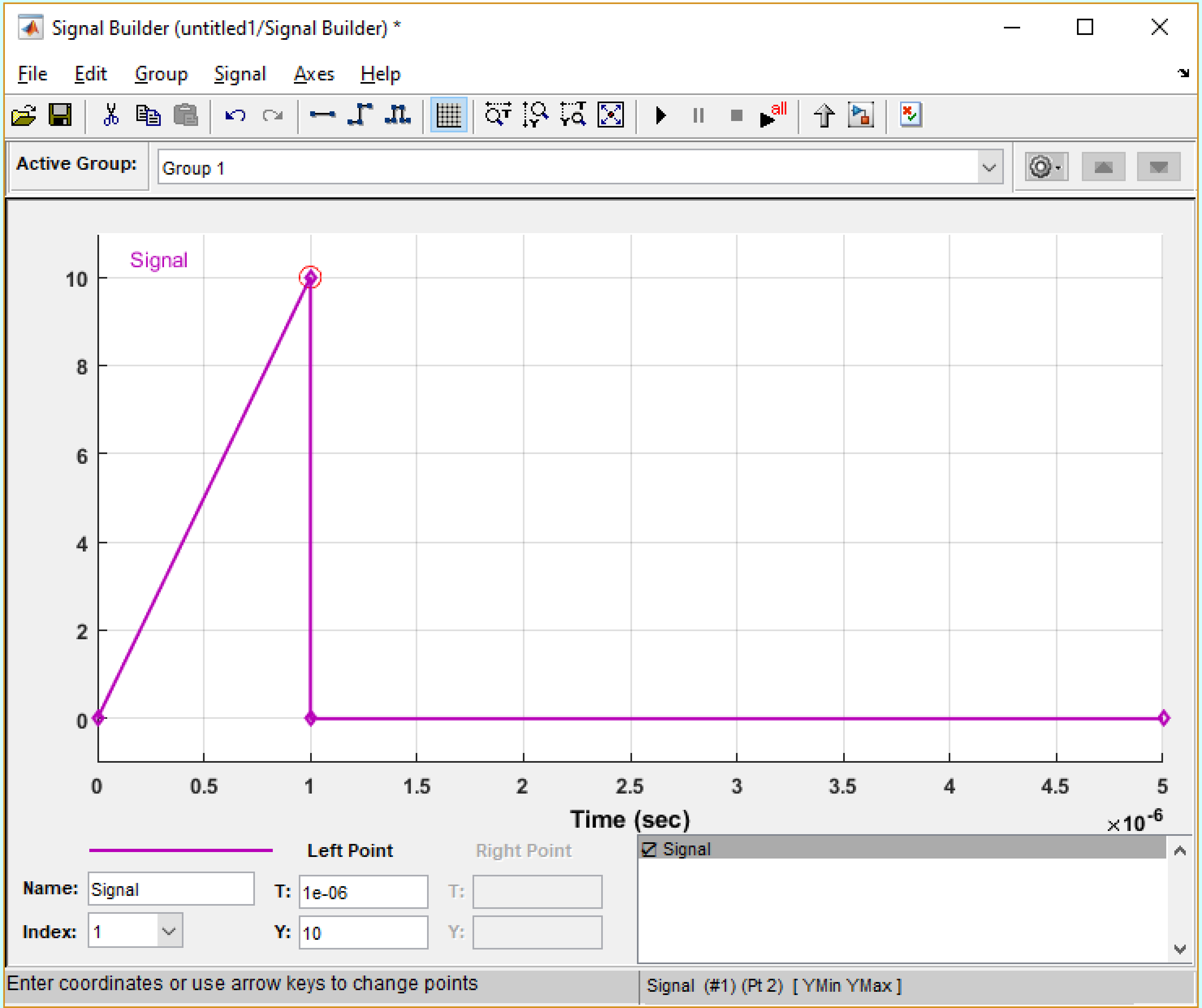
MATLAB: Simulink

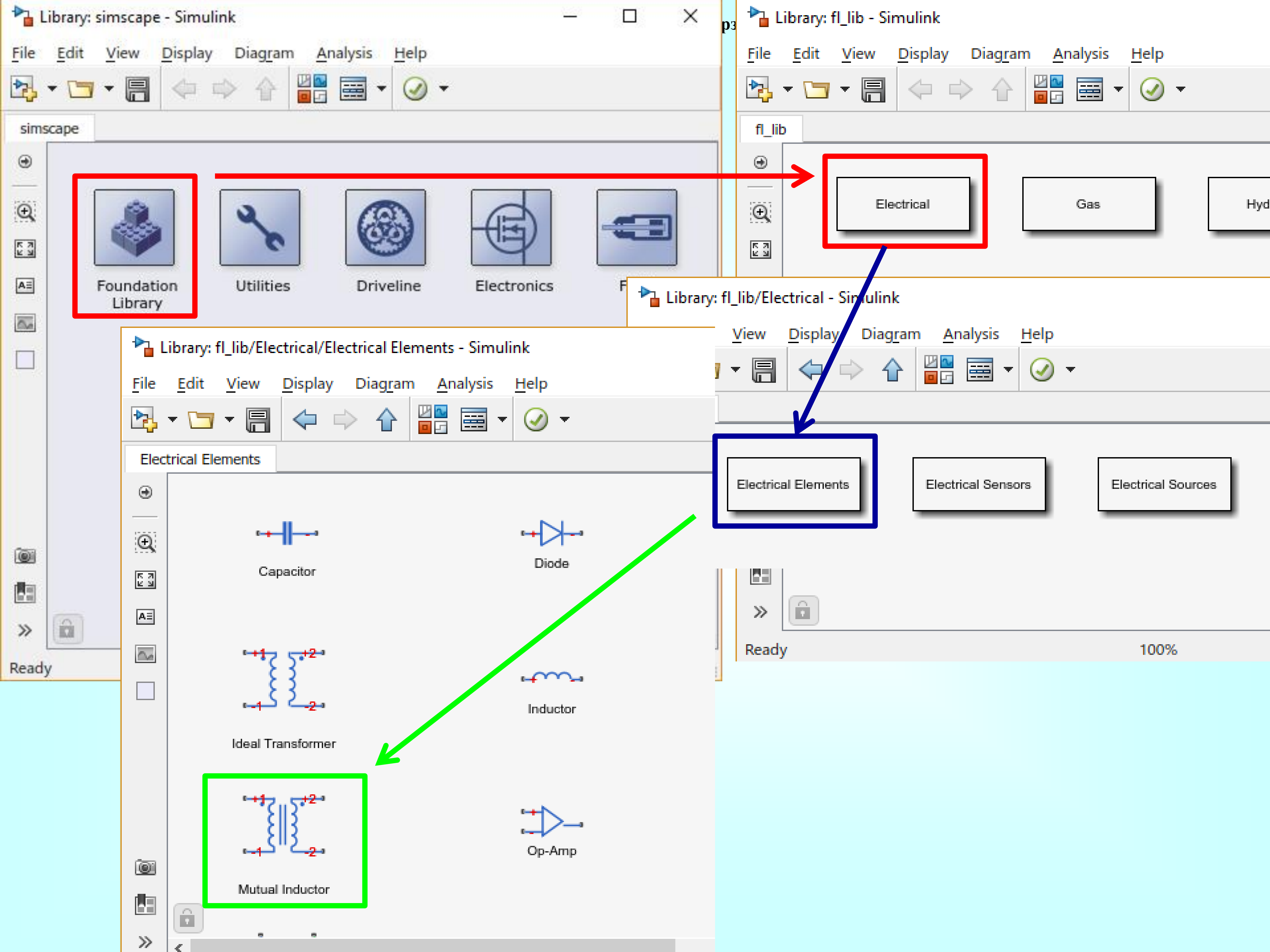
Sources



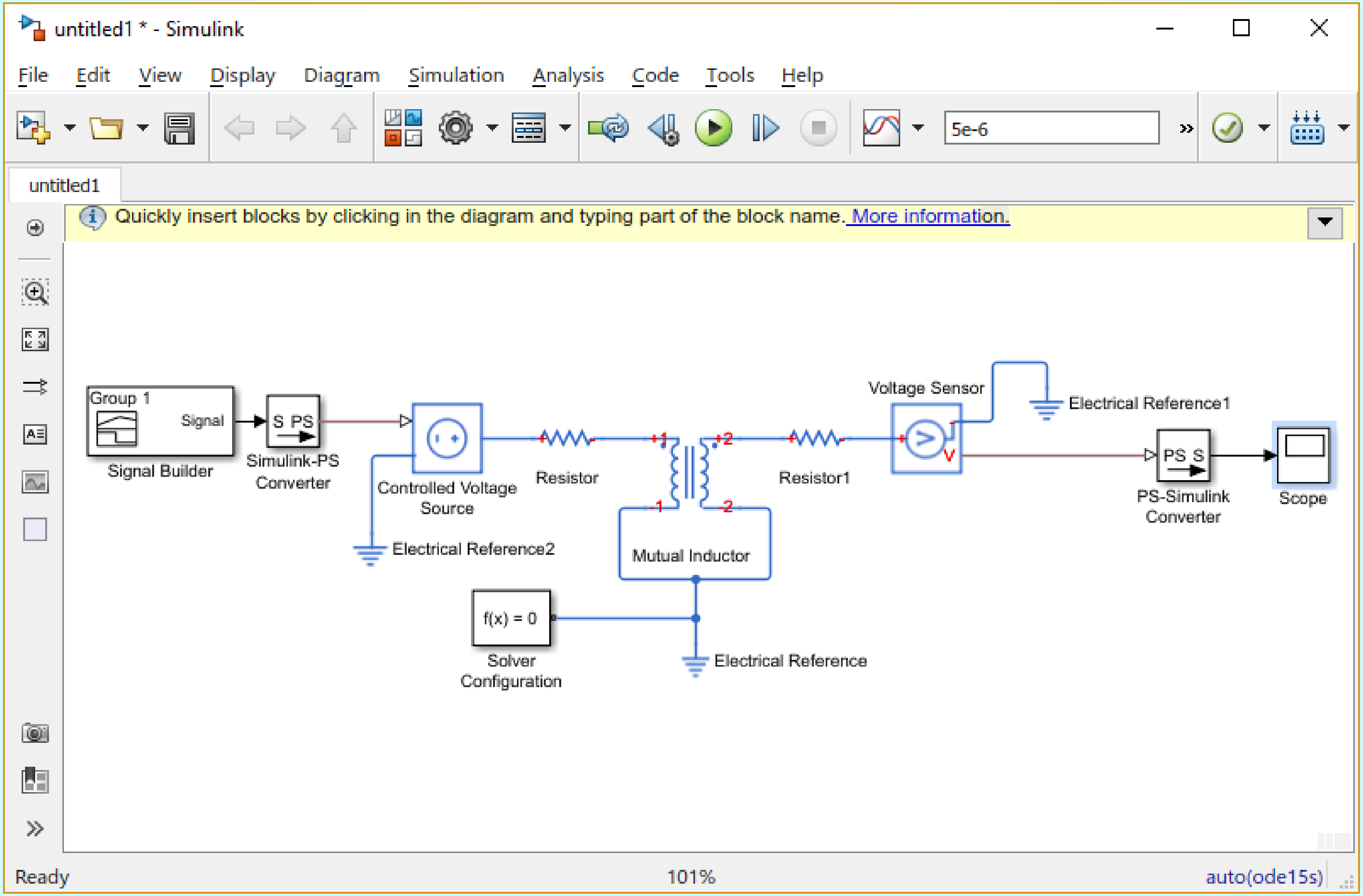


Signal Builder





Library: simscape - Simulink
File Edit View Display Diagram Analysis Help
simscape
Foundation Library
Utilities
Driveline
Electronics
Library: fl_lib - Simulink
File Edit View Display Diagram Analysis Help
fl_lib
Electrical
Gas
Library: fl_lib/Electrical - Simulink
View Display Diagram Analysis Help
Electrical Elements
Electrical Sensors
Electrical Sources
Ready
100%
Library: fl_lib/Electrical/Electrical Elements - Simulink
File Edit View Display Diagram Analysis Help
Electrical Elements
Capacitor
Diode
Ideal Transformer
Inductor
Mutual Inductor
Op-Amp
Ready


untitled1 * - Simulink
File Edit View Display Diagram Simulation Analysis Code Tools Help
5e-6
untitled1
Quickly insert blocks by clicking in the diagram and typing part of the block name. More information.
Group 1
Signal
Signal Builder
S PS
Simulink-PS Converter
Controlled Voltage Source
Electrical Reference2
Resistor
Mutual Inductor
Resistor1
Voltage Sensor
Electrical Reference1
PS S
PS-Simulink Converter
Scope
f(x) = 0
Solver Configuration
Electrical Reference
Ready
101%
auto(ode15s)

Block Parameters: Mutual Inductor

Mutual Inductor

Models a mutual inductor. If winding 1 has voltage V1 across it and current I1 flowing into its + terminal, and winding 2 has voltage V2 across it and current I2 flowing into its + terminal, then

V1 = L1*dI1/dt + M*dI2/dt

V2 = L2*dI2/dt + M*dI1/dt

where parameters L1 and L2 are the winding self-inductances, and M is the mutual inductance. M is defined in terms of the Coefficient of coupling k by M=k*sqrt(L1*L2). Hence k should be greater than zero and less than one.

Source code

Settings

Parameters | Variables

Inductance L1:	1	mH
Inductance L2:	4	mH
Coefficient of coupling:	0.5	

Block Parameters: Resistor

Resistor

The voltage-current (V-I) relationship for a linear resistor is V=I*R, where R is the constant resistance in ohms.

The positive and negative terminals of the resistor are denoted by the + and - signs respectively. By convention, the voltage across the resistor is given by V(+)-V(-), and the sign of the current is positive when flowing through the device from the positive to the negative terminal. This convention ensures that the power absorbed by a resistor is always positive.

Source code

Settings

Parameters | Variables

Resistance:	1	kOhm

Block Parameters: Resistor1

Resistor

The voltage-current (V-I) relationship

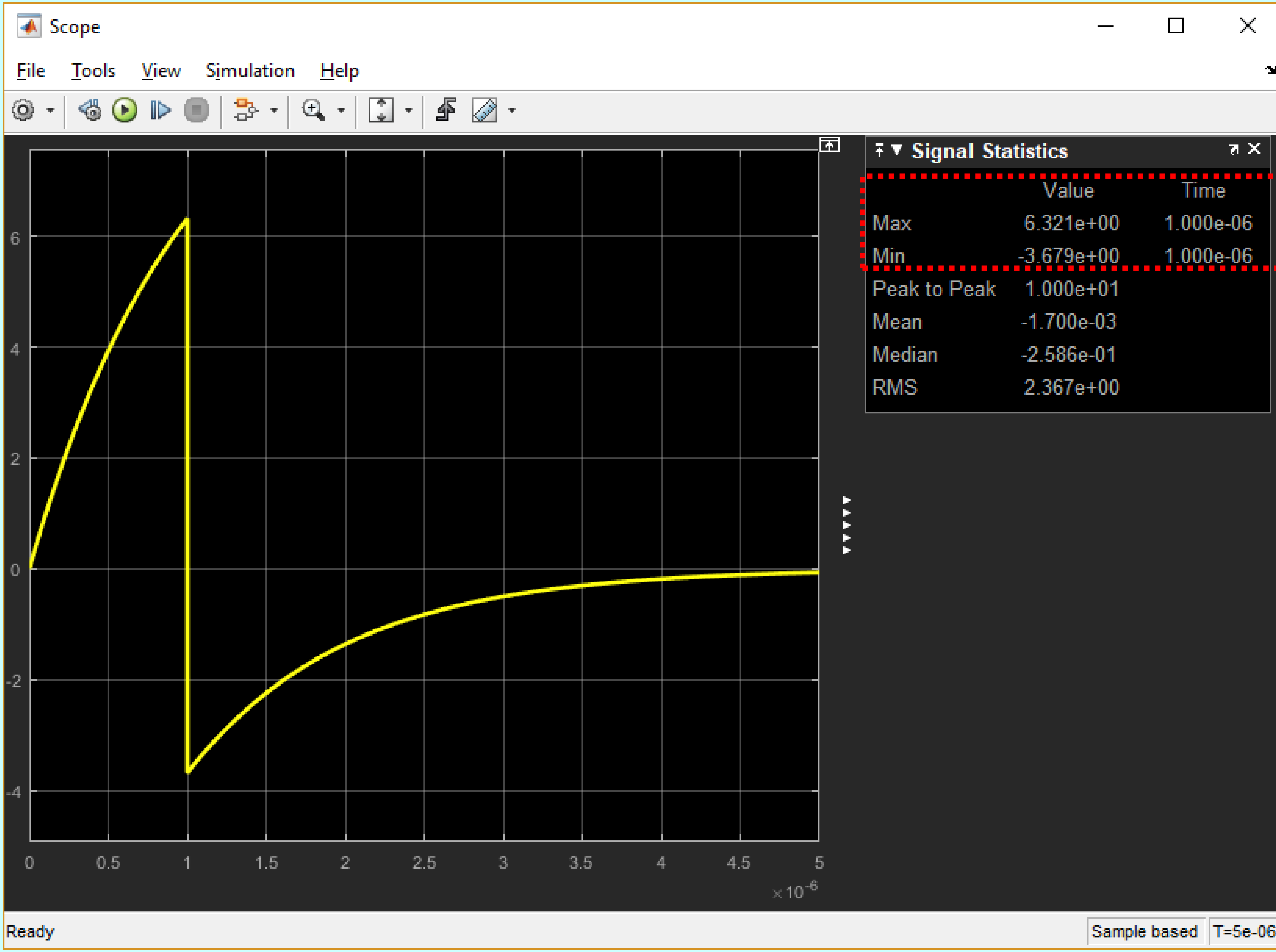
The positive and negative terminals of
the voltage across the resistor is given
the device from the positive to the ne
resistor is always positive.

Source code

Settings

Parameters | Variables

Resistance:	2	kOhm


Scope
File
Tools
View
Simulation
Help
Signal Statistics
Value
Time
Max
6.321e+00
1.000e-06
Min
-3.679e+00
1.000e-06
Peak to Peak
1.000e+01
Mean
-1.700e-03
Median
-2.586e-01
RMS
2.367e+00
6
4
2
0
-2
-4
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
×10^-6
Ready
Sample based
T=5e-06