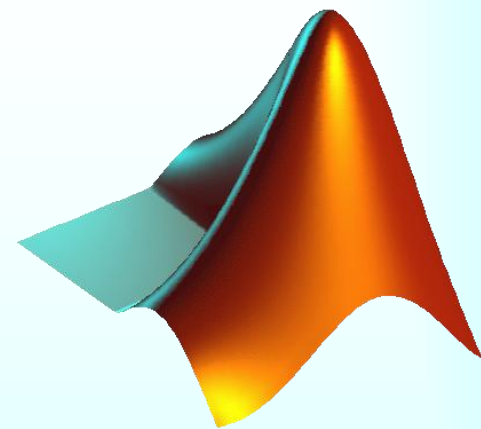
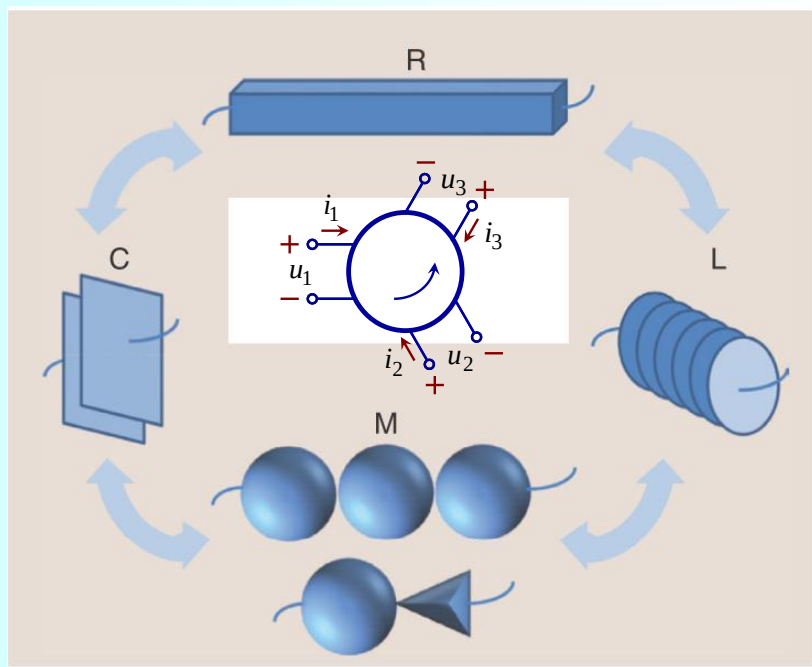


Практикум из рачунарске анализе кола



Милка Потребих Иваниш

Никола Баста



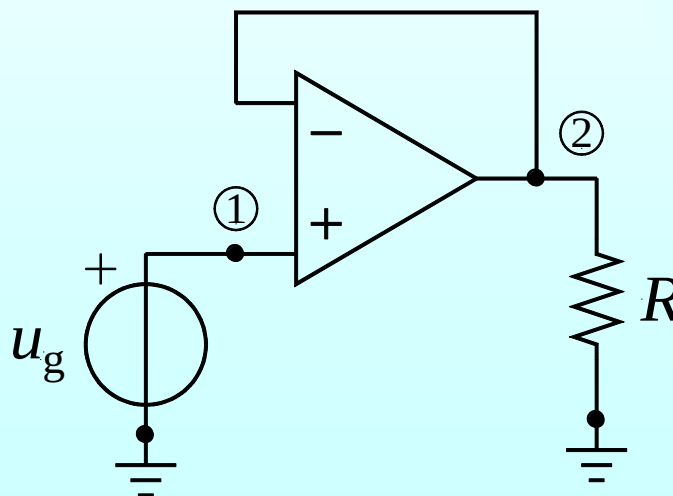
Рачунарски (софтверски) алати

- *Mathematica*, WolframAlpha, MuPAD, Maxima, SymPy, SymPy Live, SymPy Gamma
- MATLAB, Scilab, Octave, FreeMat, Julia
- LTspice, QucsStudio, ngspice, Xyce, XCircuit
- Python, MathCAD, MAPLE, GeoGebra
- Symbolab, SpeQ Mathematics, meta-calculator, Desmos, ...

Free/Libre Open Source Software (FLOSS)

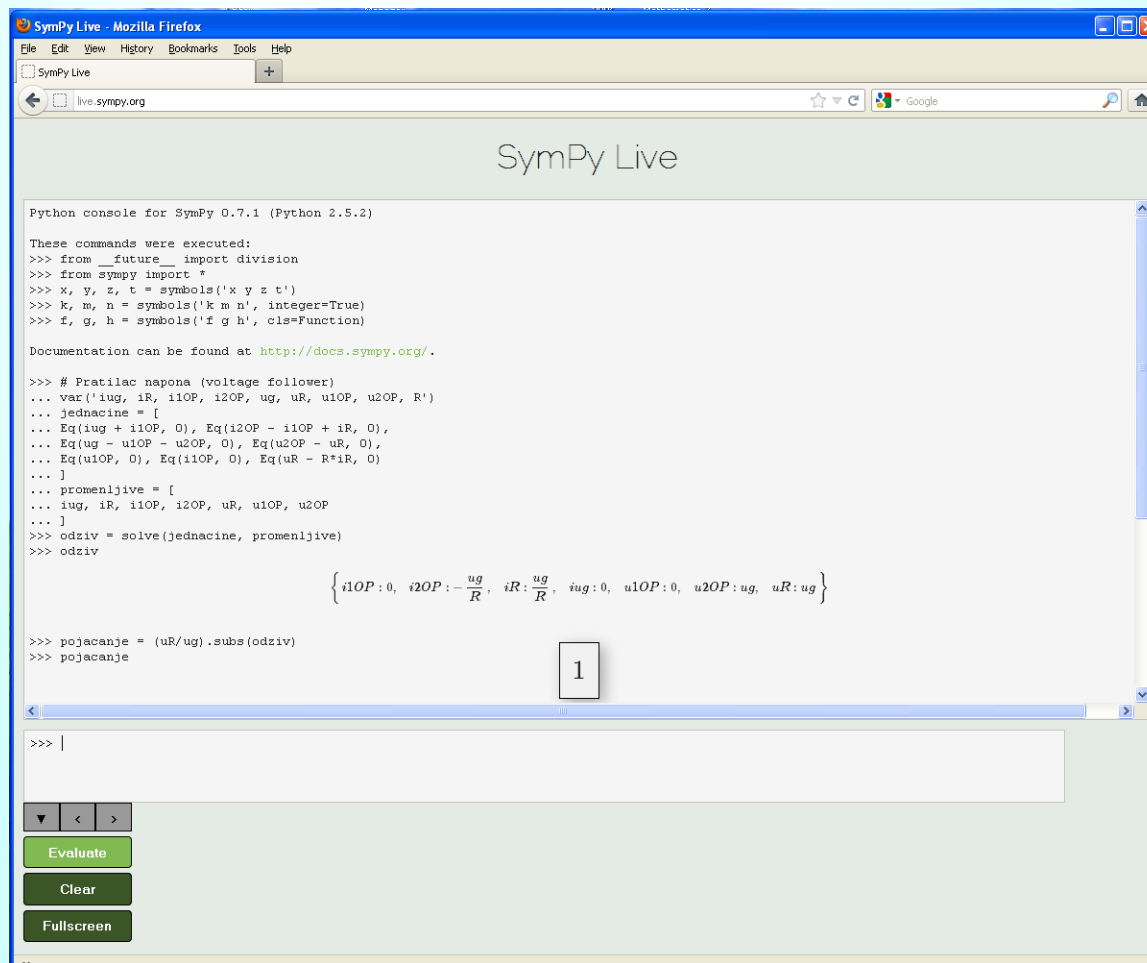
Пратилац напона

Одредити напонско појачање кола са слике, које се назива пратилац напона (voltage follower).



SymPy Live

Symbolic Python Live, <http://live.sympy.org/>



SymPy Live - Mozilla Firefox

live.sympy.org

SymPy Live

Python console for SymPy 0.7.1 (Python 2.5.2)

These commands were executed:

```
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
```

Documentation can be found at <http://docs.sympy.org/>.

```
>>> # Pratilac napona (voltage follower)
... var('iug, iR, i1OP, i2OP, ug, uR, u1OP, u2OP, R')
... jednacine = [
... Eq(iug + i1OP, 0), Eq(i2OP - i1OP + iR, 0),
... Eq(ug - u1OP - u2OP, 0), Eq(u2OP - uR, 0),
... Eq(u1OP, 0), Eq(i1OP, 0), Eq(uR - R*iR, 0)
... ]
... promenljive = [
... iug, iR, i1OP, i2OP, uR, u1OP, u2OP
... ]
>>> odziv = solve(jednacine, promenljive)
>>> odziv
```

$$\left\{ i1OP: 0, i2OP: -\frac{ug}{R}, iR: \frac{ug}{R}, iug: 0, u1OP: 0, u2OP: ug, uR: ug \right\}$$

```
>>> pojecanje = (uR/ug).subs(odziv)
>>> pojecanje
```

1

>>> |

▼ < >

Evaluate

Clear

Fullscreen

SymPy Live

Python console for SymPy 0.7.1 (Python 2.5.2)

These commands were executed:

```
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
```

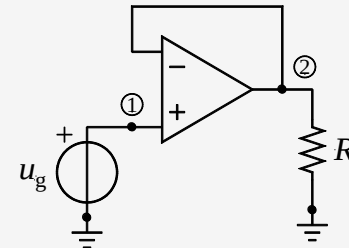
Documentation can be found at <http://docs.sympy.org/>.

```
>>> # Pratilac napona (voltage follower)
... var('iug, iR, i1OP, i2OP, ug, uR, u1OP, u2OP, R')
... jednacine = [
... Eq(iug + i1OP, 0), Eq(i2OP - i1OP + iR, 0),
... Eq(ug - u1OP - u2OP, 0), Eq(u2OP - uR, 0),
... Eq(u1OP, 0), Eq(i1OP, 0), Eq(uR - R*iR, 0)
... ]
... promenljive = [
... iug, iR, i1OP, i2OP, uR, u1OP, u2OP
... ]
>>> odz = solve(jednacine, promenljive)
>>> odz
```

$$\left\{ i1OP : 0, \quad i2OP : -\frac{ug}{R}, \quad iR : \frac{ug}{R}, \quad iug : 0, \quad u1OP : 0, \quad u2OP : ug, \quad uR : ug \right\}$$

```
>>> pojacanje = (uR/ug).subs(odz)
>>> pojacanje
```

1



```
D:\Kola\Zbirka\Latila3 Pratilac\PratilacNapona.py - Notepad++
File Edit Search View Encoding Language Settings Macro Run Plugins Window ?
PratilacNapona.py
1 # Pratilac napona (voltage follower)
2 var('iug, iR, i1OP, i2OP, ug, uR, u1OP, u2OP, R')
3 jednacine = [
4 Eq(iug + i1OP, 0), Eq(i2OP - i1OP + iR, 0),
5 Eq(ug - u1OP - u2OP, 0), Eq(u2OP - uR, 0),
6 Eq(u1OP, 0), Eq(i1OP, 0), Eq(uR - R*iR, 0)
7 ]
8 promenljive = [
9 iug, iR, i1OP, i2OP, uR, u1OP, u2OP
10 ]
11 odziv = solve(jednacine, promenljive)
12 pojacanje = (uR/ug).subs(odziv)
```

Notepad++, <http://notepad-plus-plus.org/>

MATLAB: Symbolic Math Toolbox

```
clear variables

syms iug iR i1OP i2OP ug uR u1OP u2OP R
promenljive = [iug, iR, i1OP, i2OP, uR, u1OP, u2OP]

promenljive = (iug iR i1OP i2OP uR u1OP u2OP)

jednacine = [iug + i1OP == 0, i2OP - i1OP + iR == 0, ug - u1OP - u2OP == 0, ...
             u2OP - uR == 0, u1OP == 0, i1OP == 0, uR == R*iR]

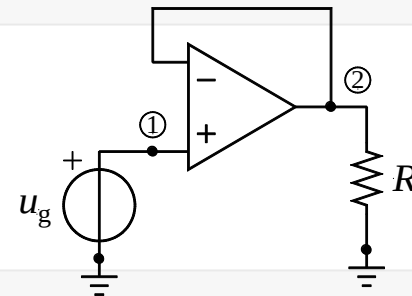
jednacine = (i1OP + iug = 0 i2OP - i1OP + iR = 0 ug - u2OP - u1OP = 0 u2OP - uR = 0 u1OP = 0 i1OP = 0 uR = R*iR)

odziv = solve(jednacine, promenljive)

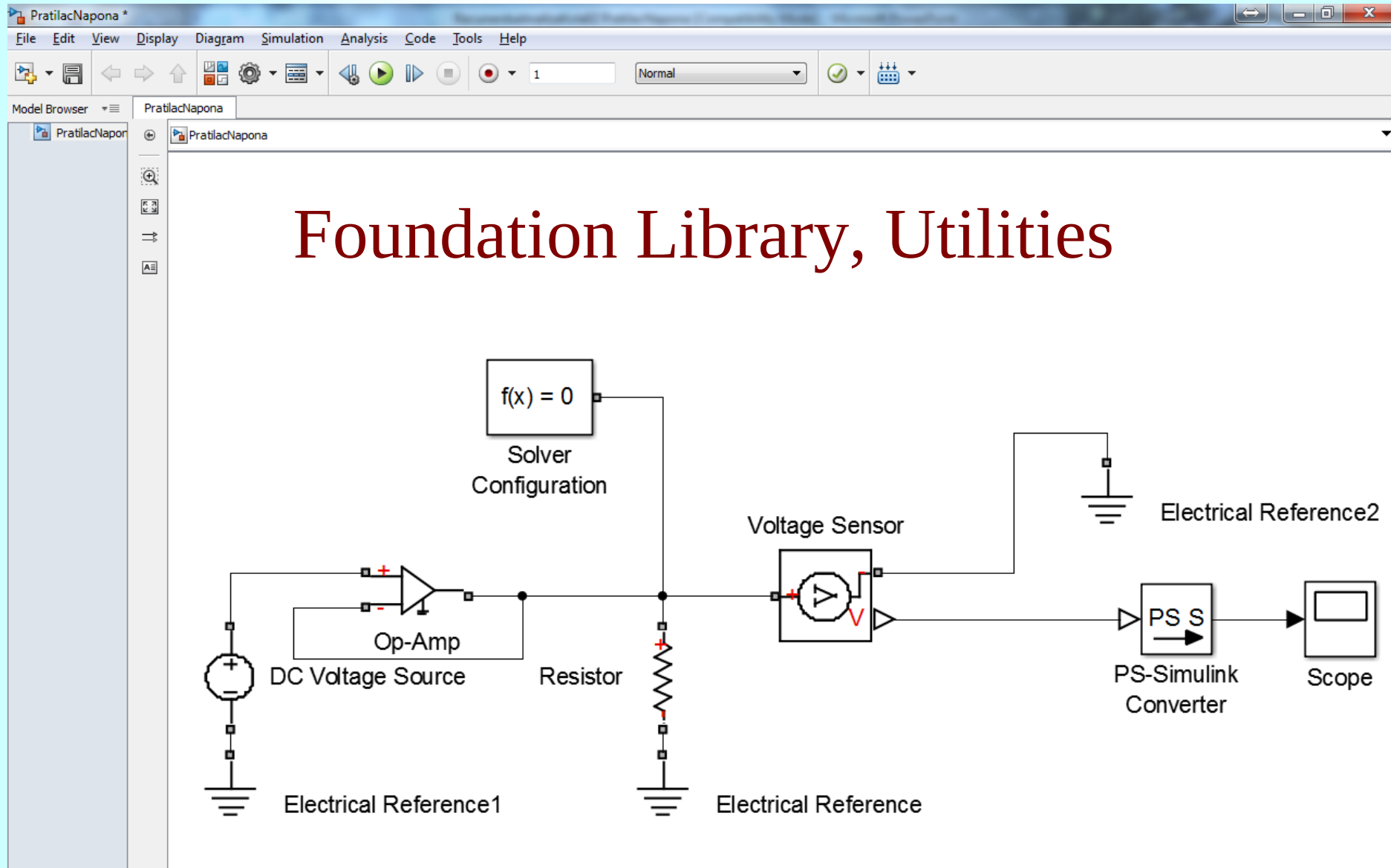
odziv = struct with fields:
    iug: 0
    iR: ug/R
    i1OP: 0
    i2OP: -ug/R
    uR: ug
    u1OP: 0
    u2OP: ug

pojacanje = odziv.u2OP / ug

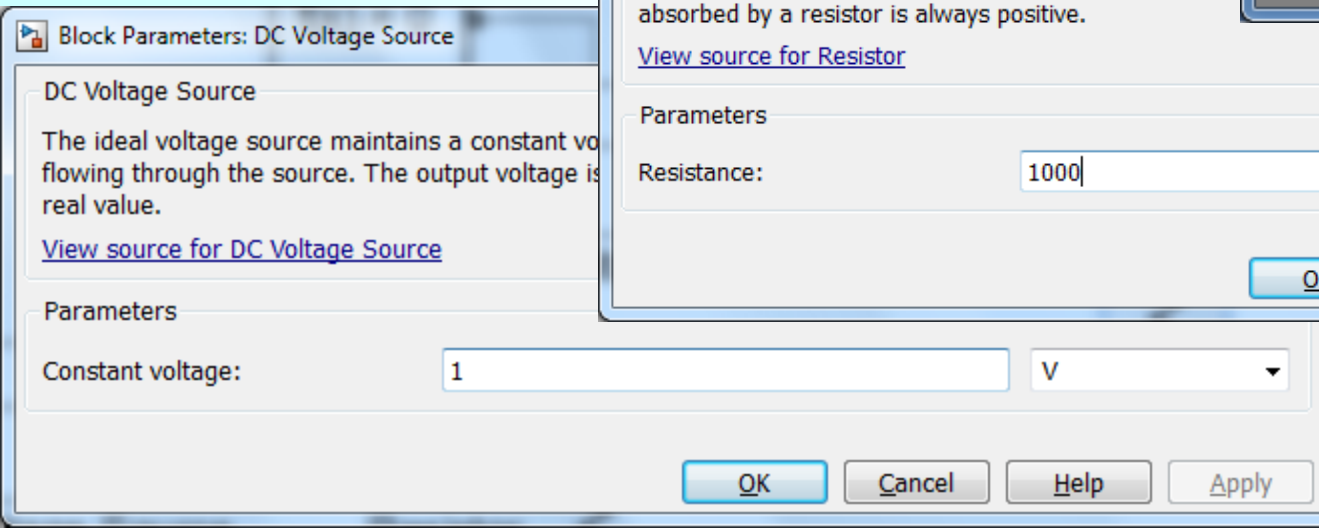
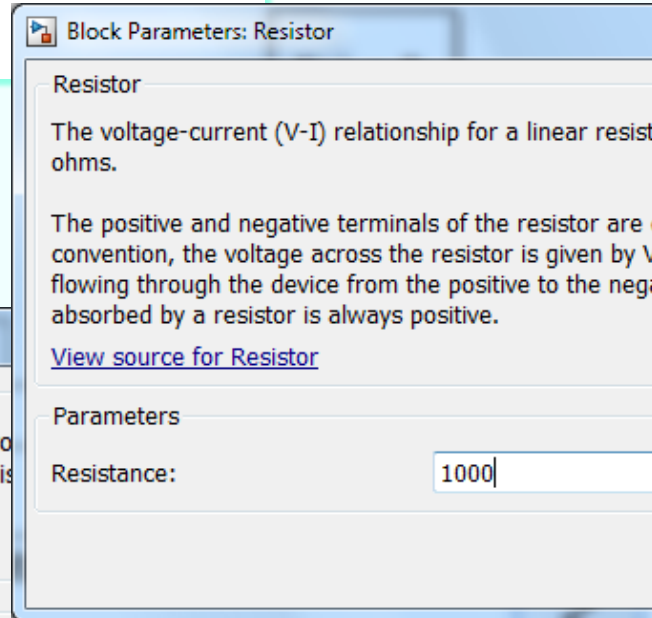
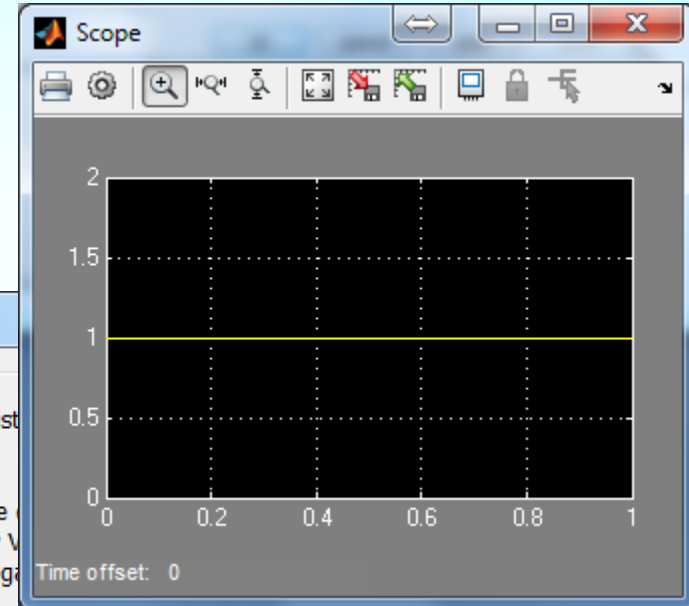
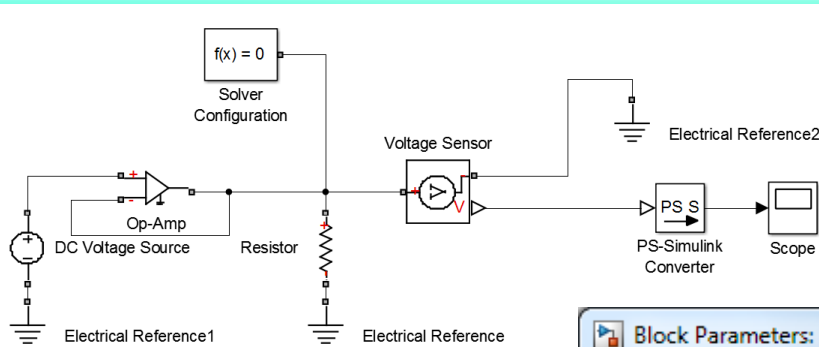
pojacanje = 1
```



MATLAB: Simscape



MATLAB: Simscape Foundation Library, Utilities



Maxima

Maxima: <http://maxima.sourceforge.net/>

wxMaxima: <http://andrejv.github.com/wxmaxima/index.html>

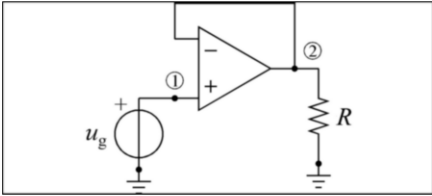
CESGA Maxima on line: <http://maxima.cesga.es/>

wxMaxima 12.01.0 [PratilacNapona.wxmx]

File Edit Cell Maxima Equations Algebra Calculus Simplify Plot Numeric Help

Pratilac napona (voltage follower)

Figure 1:



```
(%i1) jednacine: [iug + i1OP = 0, i2OP - i1OP + iR = 0,
ug - u1OP - u2OP = 0, u2OP - uR = 0,
u1OP = 0, i1OP = 0, uR = R*iR];

(%o1) [i1OP+iug=0,iR+i2OP-i1OP=0,-u2OP-u1OP+ug=0,u2OP-uR
=0,u1OP=0,i1OP=0,uR=iRR]

(%i2) odziv: linsolve(jednacine,
[iug, iR, i1OP, i2OP, uR, u1OP, u2OP]);

(%o2) [iug=0,iR=ug/R,i1OP=0,i2OP=-ug/R,uR=ug,u1OP=0,u2OP=ug]

(%i3) pojacanje: uR/ug, odziv;

(%o3) 1
```

Ready for user input

Инструментациони појачавач

- Одредити напон v_3 отпорника R_8 ?
- Нацртати график напона v_3 отпорника R_8 у функцији времена?
- Колика је снага извора?

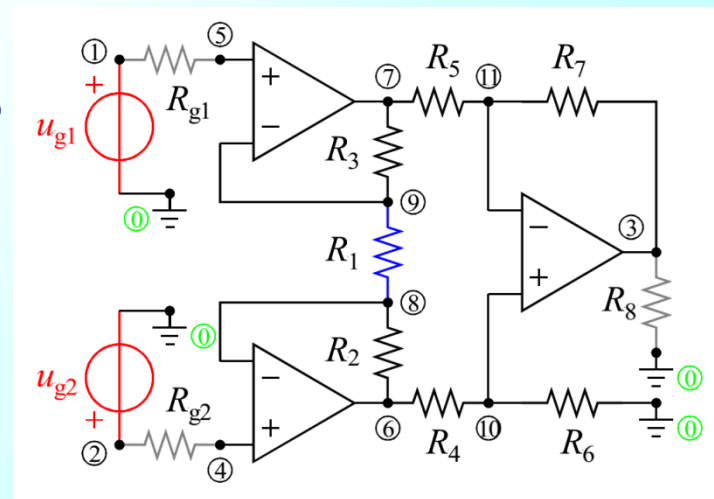
Вредности елемената су познате:

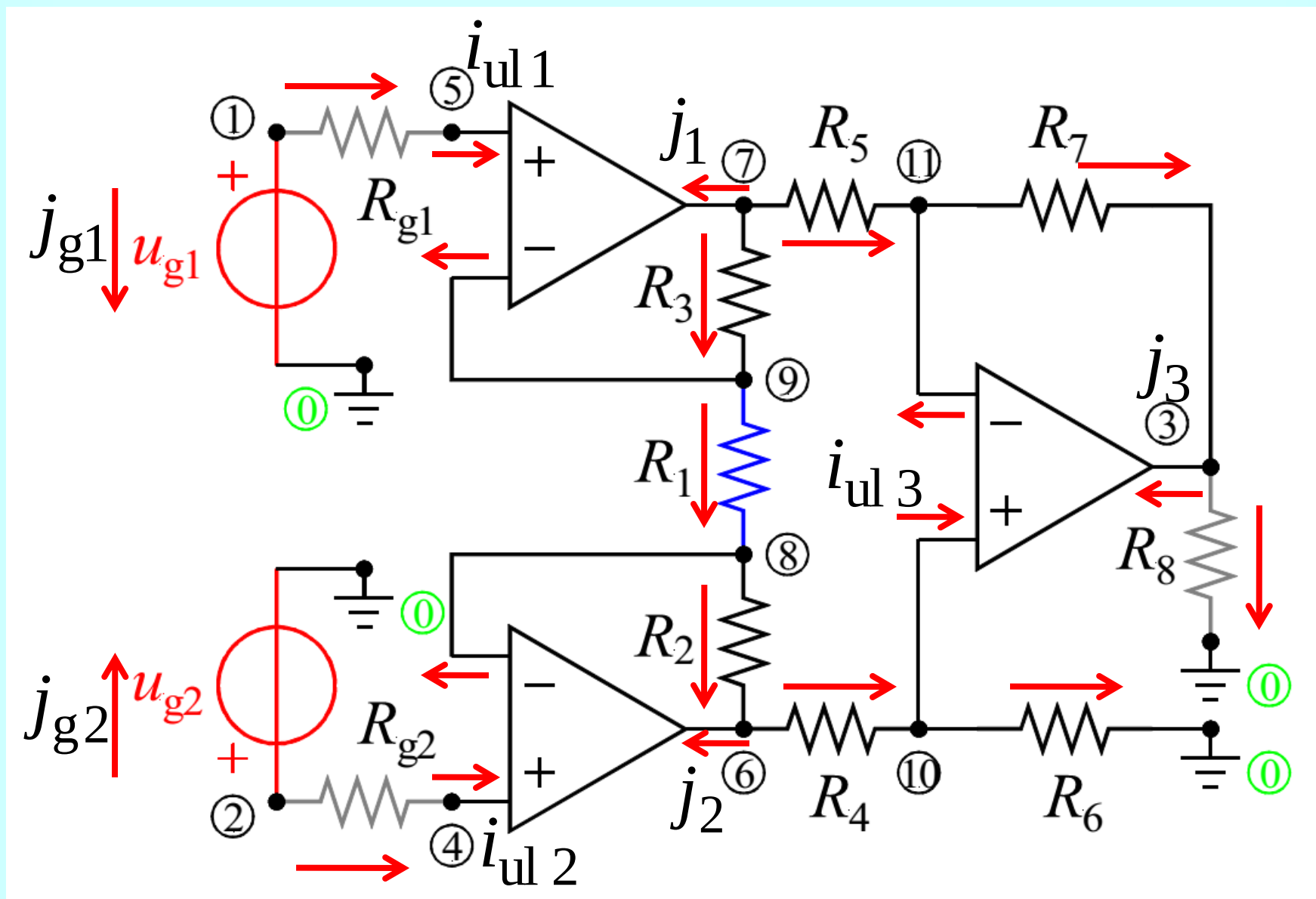
$$R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = R$$

$$R = 1 \text{ k}\Omega, R_1 = 2 \text{ k}\Omega, R_{g1} = R_{g2} = 1 \text{ k}\Omega$$

$$u_{g1}(t) = U h(t), u_{g2}(t) = U h(t-T)$$

$$U = 10 \text{ V}, T = 1 \text{ s}$$





Модификована метода потенцијала чворова

$$(1) j_{g1} + (V_1 - V_5)/R_{g1} = 0$$

$$(2) j_{g2} + (V_2 - V_4)/R_{g2} = 0$$

$$(3) j_3 - (V_{11} - V_3)/R_7 + V_3/R_8 = 0$$

$$(4) -(V_2 - V_4)/R_{g2} + i_{ul2} = 0$$

$$(5) -(V_1 - V_5)/R_{g1} + i_{ul1} = 0$$

$$(6) j_2 + (V_6 - V_{10})/R_4 - (V_8 - V_6)/R_2 = 0$$

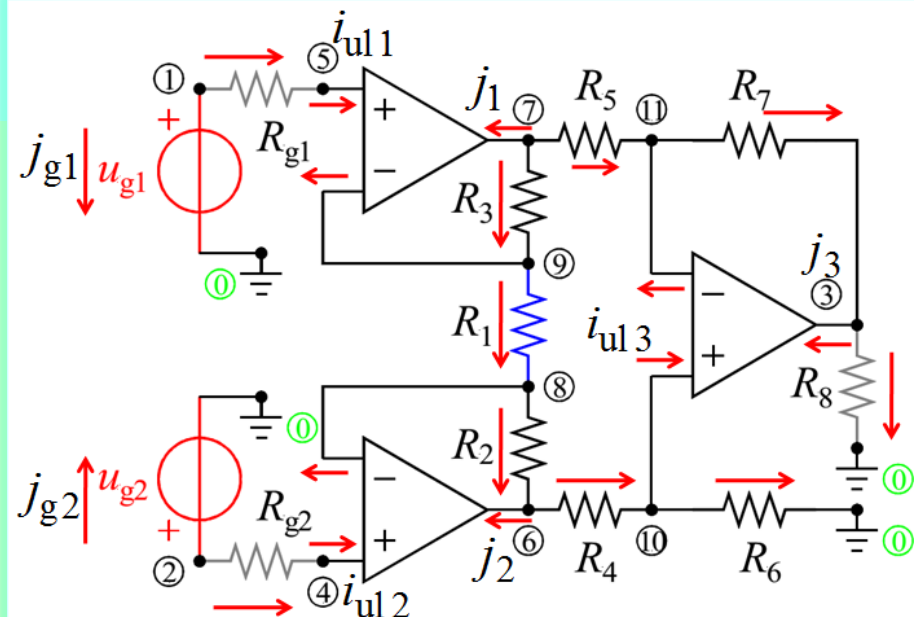
$$(7) j_1 + (V_7 - V_9)/R_3 + (V_7 - V_{11})/R_5 = 0$$

$$(8) -i_{ul2} - (V_9 - V_8)/R_1 + (V_8 - V_6)/R_2 = 0$$

$$(9) -i_{ul1} - (V_7 - V_9)/R_3 + (V_9 - V_8)/R_1 = 0$$

$$(10) -(V_6 - V_{10})/R_4 + i_{ul3} + V_{10}/R_6 = 0$$

$$(11) -(V_7 - V_{11})/R_5 - i_{ul3} + (V_{11} - V_3)/R_7 = 0$$



$$i_{ul1} = 0$$

$$i_{ul2} = 0$$

$$i_{ul3} = 0$$

$$V_5 = V_9$$

$$V_4 = V_8$$

$$V_{10} = V_{11}$$

$$V_1 = u_{g1}$$

$$V_2 = u_{g2}$$

MATLAB: Symbolic Math Toolbox

Променљиве, једначине, решавање

```
clear variables
```

```

syms R R1 R2 R3 R4 R5 R6 R7 R8 Rg1 Rg2 iul1 iul2 iul3 j1 j2 j3 jg1 jg2 ug1 ug2 v1 v10 v11 v2 v3 v4 v5 v6 v7 v8 v9
jednacie = unique([jg1 + (v1 - v5)/Rg1 == sym(0),...
    v1 == ug1, jg2 + (v2 - v4)/Rg2 == sym(0),...
    v2 == ug2, j3 + v3/R8 - (v11 - v3)/R7 == sym(0),...
    -((v2 - v4)/Rg2) + iul2 == sym(0), -((v1 - v5)/Rg1) + iul1 == sym(0),...
    j2 + (v6 - v10)/R4 - (v8 - v6)/R2 == sym(0), j1 + (v7 - v11)/R5 + (v7 - v9)/R3 == sym(0), ...
    -iul2 - (v9 - v8)/R1 + (v8 - v6)/R2 == sym(0), -iul1 - (v7 - v9)/R3 + (v9 - v8)/R1 == sym(0),...
    -((v6 - v10)/R4) + v10/R6 + iul3 == sym(0), -((v7 - v11)/R5) + (v11 - v3)/R7 - iul3 == sym(0),...
    iul1 == sym(0), iul2 == sym(0), iul3 == sym(0), ...
    v5 == v9, v4 == v8, v10 == v11])

```

syms

sym

jednacine =

$$\left(v_1 = u g_1 \quad v_2 = u g_2 \quad v_4 = v_8 \quad v_5 = v_9 \quad v_{10} = v_{11} \quad i u l_3 + \frac{v_{10}}{R_6} - \frac{v_6 - v_{10}}{R_4} = 0 \quad j_3 + \frac{v_3}{R_8} + \frac{v_3 - v_{11}}{R_7} = 0 \quad j_2 + \frac{v_6 - v_8}{R_2} + \frac{v_6 - v_{10}}{R_4} = 0 \quad j_1 + \frac{v_7 - v_9}{R_3} + \frac{v_7 - v_{11}}{R_5} = 0 \quad i u l_1 = 0 \quad i u l_2 = 0 \quad i u l_3 = 0 \quad i u l_1 - \frac{v_1 - v_5}{R g_1} = 0 \quad i u \right.$$

```
assume( $\theta < Rg1 \ \& \ \theta < Rg2 \ \& \ \theta < R1 \ \& \ \theta < R2 \ \& \ \theta < R3 \ \& \ \theta < R4 \ \& \ \theta < R5 \ \& \ \theta < R6 \ \& \ \theta < R7 \ \& \ \theta < R8 \ \& \ \theta < R$ )
```

assume

```
promenljive = [v1, v2, v3, v4, v5, v6, v7, v8, v9, v10, v11, j1, j2, j3, iul1, iul2, iul3, jg1, jg2]
```

```
promenljive = (v1 v2 v3 v4 v5 v6 v7 v8 v9 v10 v11 j1 j2 j3 iul1 iul2 iul3 jg1 jg2)
```

solve

```
odziv = solve(jednacine, promenljive)
```

odziv = struct with fields:

```
v1: ug1
v2: ug2
v3: -(R1*R4*R7*ug1 - R1*R5*R6*ug2 + R2*R5*R6*ug1 + R1*R6*R7*ug1 - R2*R5*R6*ug2 + R3*R4*R7*ug1 - R1*R6*R7*ug2 + R2*R6*R7*ug1 - R3*R4*R7*ug2 - R2*R6*R7*ug2 + R3*R6*R7*ug1 - R3*R6*R7*ug2)/(R1*R4 + R1*R5 + R2*R4 + R2*R5 + R3*R4 + R3*R5 + R4*R5 + R4*R6 + R5*R6 + R5*R7 + R6*R7)
v4: ug2
v5: ug1
v6: (R1*ug2 - R2*ug1 + R2*ug2)/R1
v7: (R1*ug1 + R3*ug1 - R3*ug2)/R1
v8: ug2
v9: ug1
v10: (R6*(R1*ug2 - R2*ug1 + R2*ug2))/(R1*(R4 + R6))
v11: (R6*(R1*ug2 - R2*ug1 + R2*ug2))/(R1*(R4 + R6))
j1: -(R1*R4*ug1 + R1*R6*ug1 + R3*R4*ug1 - R1*R6*ug2 + R2*R6*ug1 - R3*R4*ug2 - R2*R6*ug2 + R3*R6*ug1 + R4*R5*ug1 - R3*R6*ug2 - R4*R5*ug2 + R5*R6*ug1 - R5*R6*ug2)/(R1*R5*(R4 + R6))
j2: -(R1*ug2 - R2*ug1 + R2*ug2 - R4*ug1 + R4*ug2 - R6*ug1 + R6*ug2)/(R1*(R4 + R6))
j3: (R1*R4*R7*ug1 + R1*R4*R8*ug1 - R1*R5*R6*ug2 + R2*R5*R6*ug1 + R1*R6*R7*ug1 - R2*R5*R6*ug2 + R3*R4*R7*ug1 - R1*R6*R7*ug2 + R1*R6*R8*ug1 + R2*R6*R7*ug1 - R3*R4*R7*ug2 + R3*R4*R8*ug1 - R1*R5*R6*ug1 + R1*R5*R6*ug2)/(R1*(R4 + R6))
iu1: 0
iu2: 0
iu3: 0
jg1: 0
ie2: 0
```

MATLAB: Symbolic Math Toolbox

Замена променљивих и вредности

```
zamena = [R2 == R, R3 == R, R4 == R, R5 == R, R6 == R, R7 == R]
```

```
zamena = (R2 = R R3 = R R4 = R R5 = R R6 = R R7 = R)
```

```
odziv.v3
```

```
ans =
```

$$\frac{R_1 R_4 R_7 u_{g1} - R_1 R_5 R_6 u_{g2} + R_2 R_5 R_6 u_{g1} + R_1 R_6 R_7 u_{g1} - R_2 R_5 R_6 u_{g2} + R_3 R_4 R_7 u_{g1} - R_1 R_6 R_7 u_{g2} + R_2 R_6 R_7 u_{g1} - R_3 R_4 R_7 u_{g2} - R_2 R_6 R_7 u_{g2} + R_3 R_6 R_7 u_{g1} - R_3 R_6 R_7 u_{g2}}{R_1 R_5 (R_4 + R_6)}$$

subs

```
subs(odziv.v3, lhs(zamena), rhs(zamena))
```

```
ans =
```

$$\frac{4 R^3 u_{g1} - 4 R^3 u_{g2} + 2 R^2 R_1 u_{g1} - 2 R^2 R_1 u_{g2}}{2 R^2 R_1}$$

simplify

```
v3Sol = simplify(subs(odziv.v3, lhs(zamena), rhs(zamena)))
```

```
v3Sol =
```

$$\frac{(2 R + R_1) (u_{g1} - u_{g2})}{R_1}$$

```
syms T U t uR8(t) ug1(t) ug2(t)
assume(0 < U & 0 < T & 0 < t);
pobude = [ug1 == U * heaviside(t-T), ug2 == U * heaviside(t - 2*T)]
```

```
pobude(t) = (ug1(t) = U heaviside(t - T) ug2(t) = U heaviside(t - 2 T))
```

```
subs(v3Sol, lhs(pobude), rhs(pobude))
```

```
ans =
```

$$\frac{(2 R + R_1) (U \text{heaviside}(t - T) - U \text{heaviside}(t - 2 T))}{R_1}$$

MATLAB: Symbolic Math Toolbox

График решења

```
syms T U t uR8(t) ug1(t) ug2(t)
assume(0 < U & 0 < T & 0 < t);
pobude = [ug1 == U * heaviside(t-T), ug2 == U * heaviside(t - 2*T)]

pobude(t) = (ug1(t) = U heaviside(t - T) ug2(t) = U heaviside(t - 2 T))

|
subs(v3Sol, lhs(pobude), rhs(pobude))

ans =
- (2 R + R1) (U heaviside(t - T) - U heaviside(t - 2 T))
R1

vrednosti = [U == sym(10), T == sym(1), R == sym(1000), R1 == sym(2000)]

vrednosti = (U = 10 T = 1 R = 1000 R1 = 2000)

uR8(t) = subs(subs(v3Sol, lhs(pobude), rhs(pobude)), lhs(vrednosti), rhs(vrednosti))

uR8(t) = 20 heaviside(t - 2) - 20 heaviside(t - 1)

ug1(t) = subs(subs(ug1, lhs(pobude), rhs(pobude)), lhs(vrednosti), rhs(vrednosti))

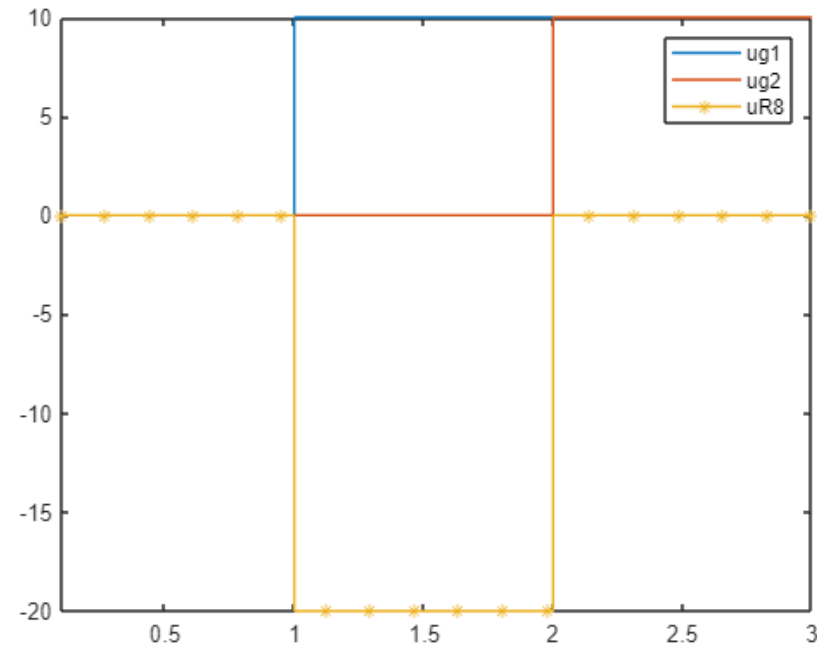
ug1(t) = 10 heaviside(t - 1)

ug2(t) = subs(subs(ug2, lhs(pobude), rhs(pobude)), lhs(vrednosti), rhs(vrednosti))

ug2(t) = 10 heaviside(t - 2)

figure
fplot(ug1(t), [0.1, 3])
hold on
fplot(ug2(t), [0.1, 3])
fplot(uR8(t), [0.1, 3], '-*')
hold off
legend({'ug1', 'ug2', 'uR8'});
```

figure
fplot

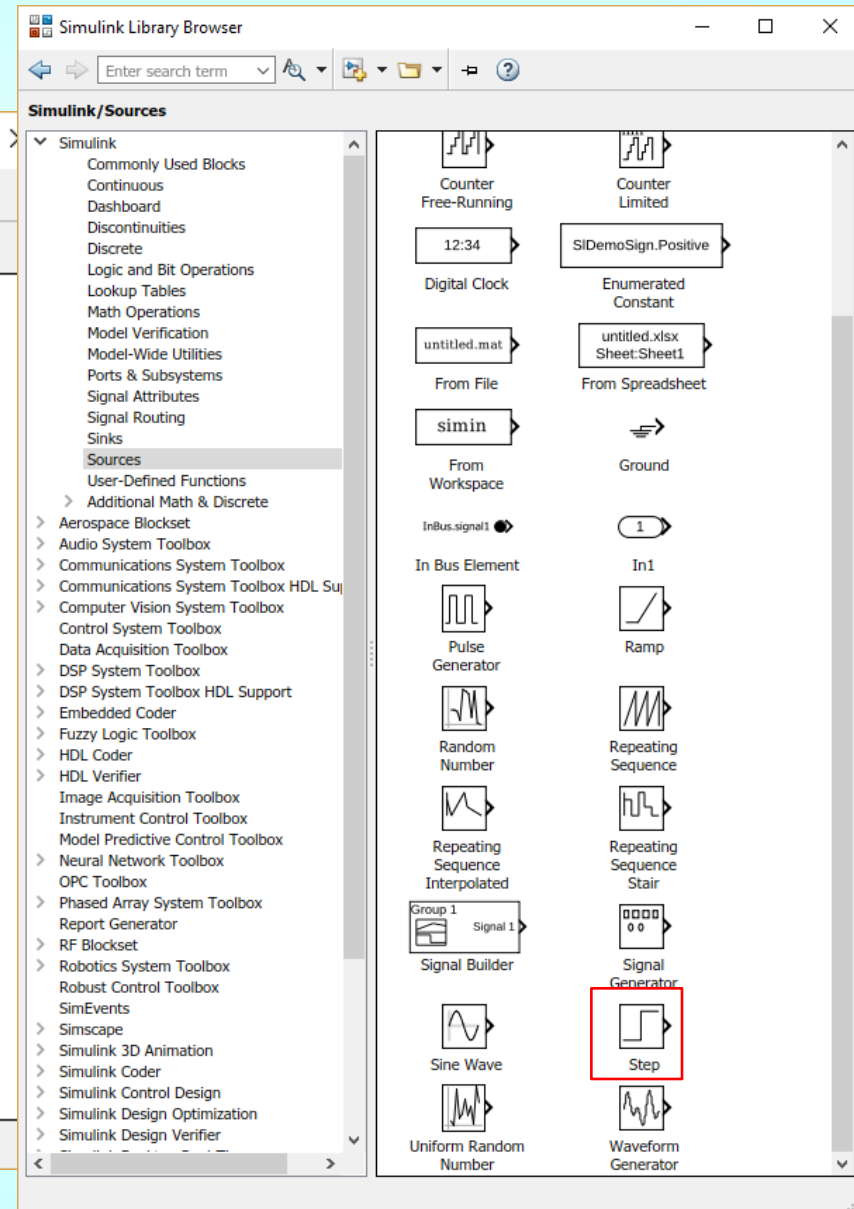
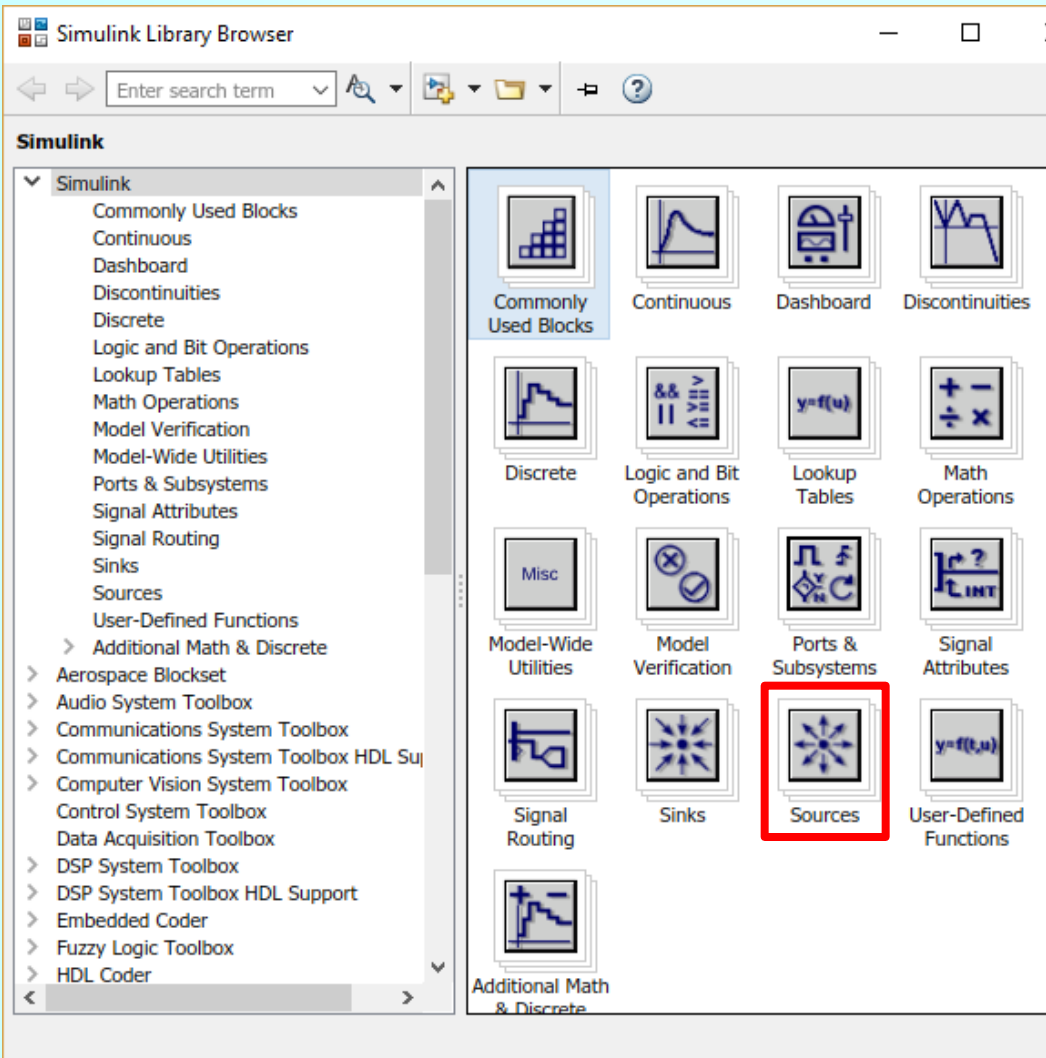


```
pg = subs(jg1*ug1 + jg2*ug2, odziv)
```

```
pg(t) = 0
```

MATLAB: Simulink

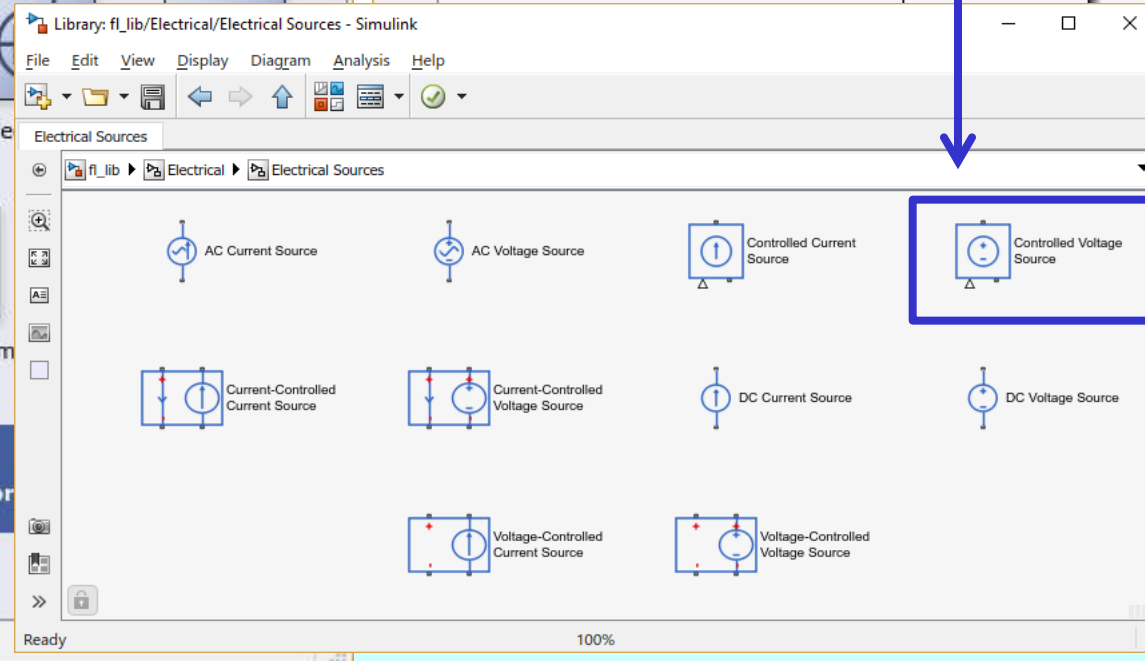
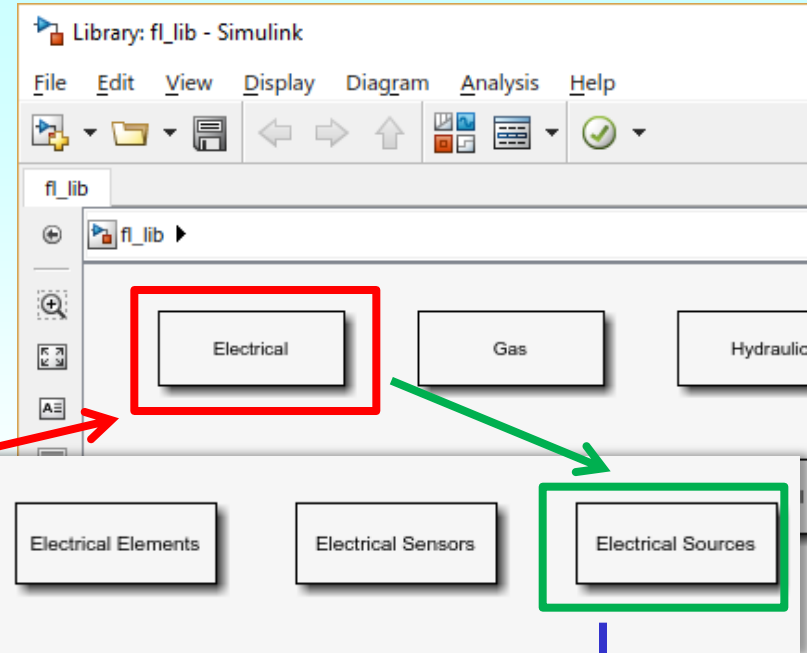
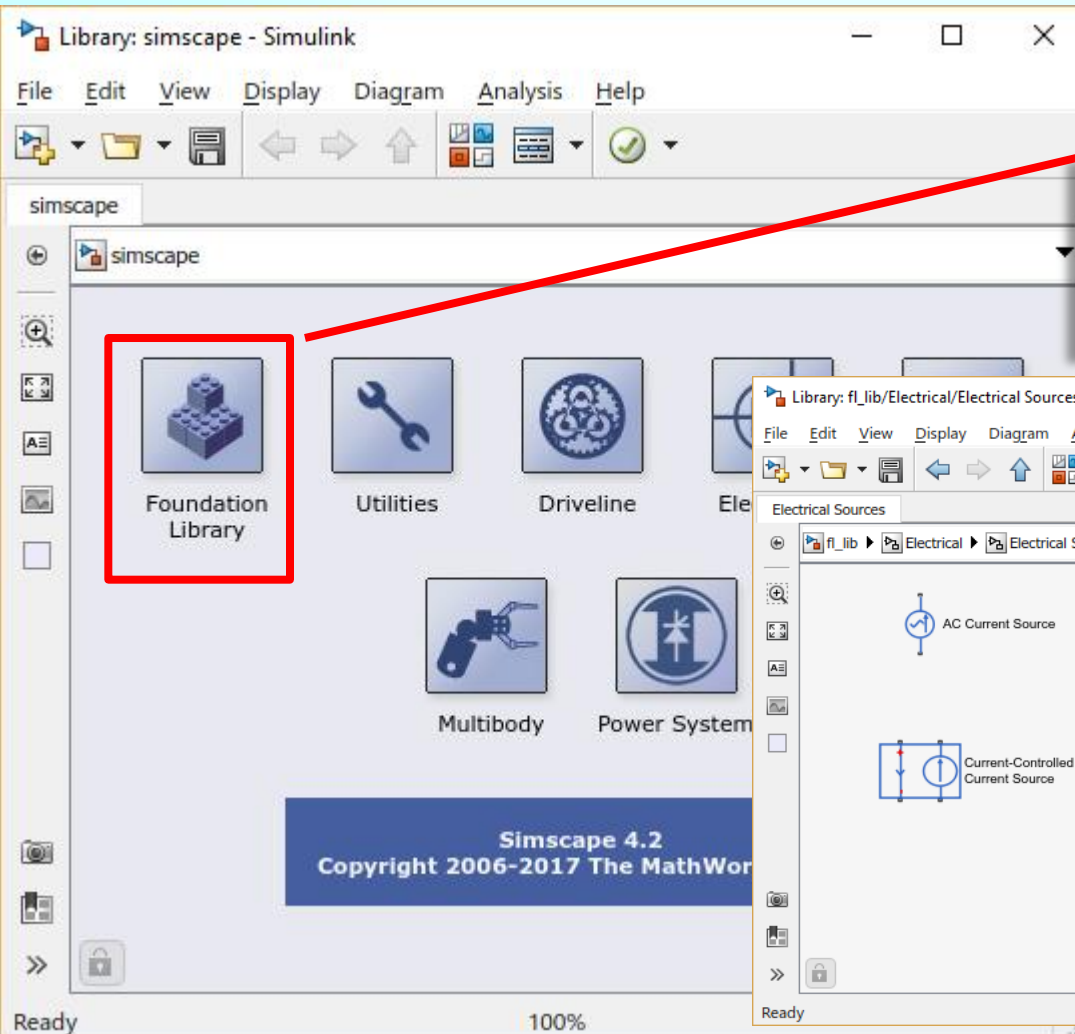
Sources



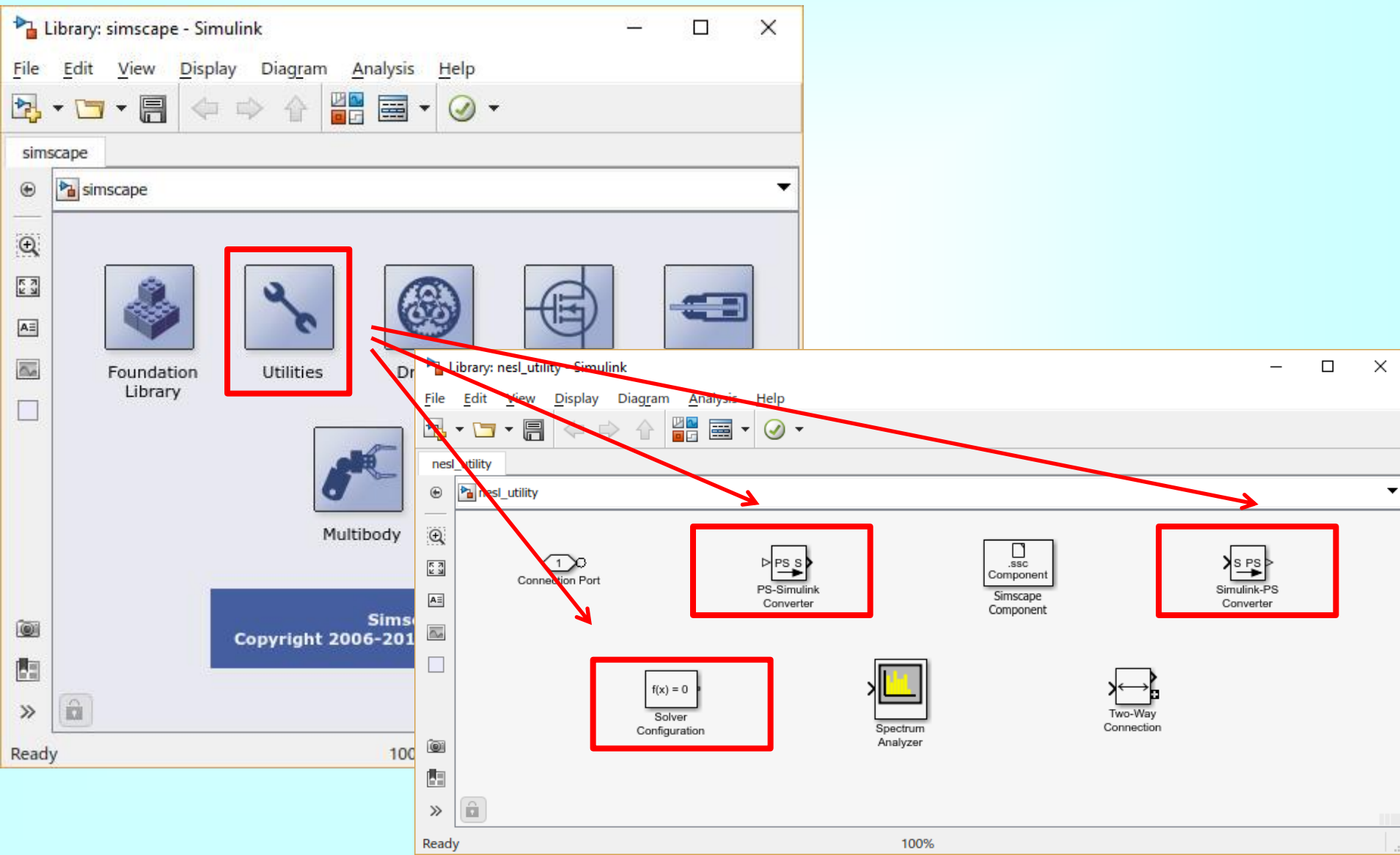
MATLAB: Simscape

Foundation Library

Electrical, Electrical Sources

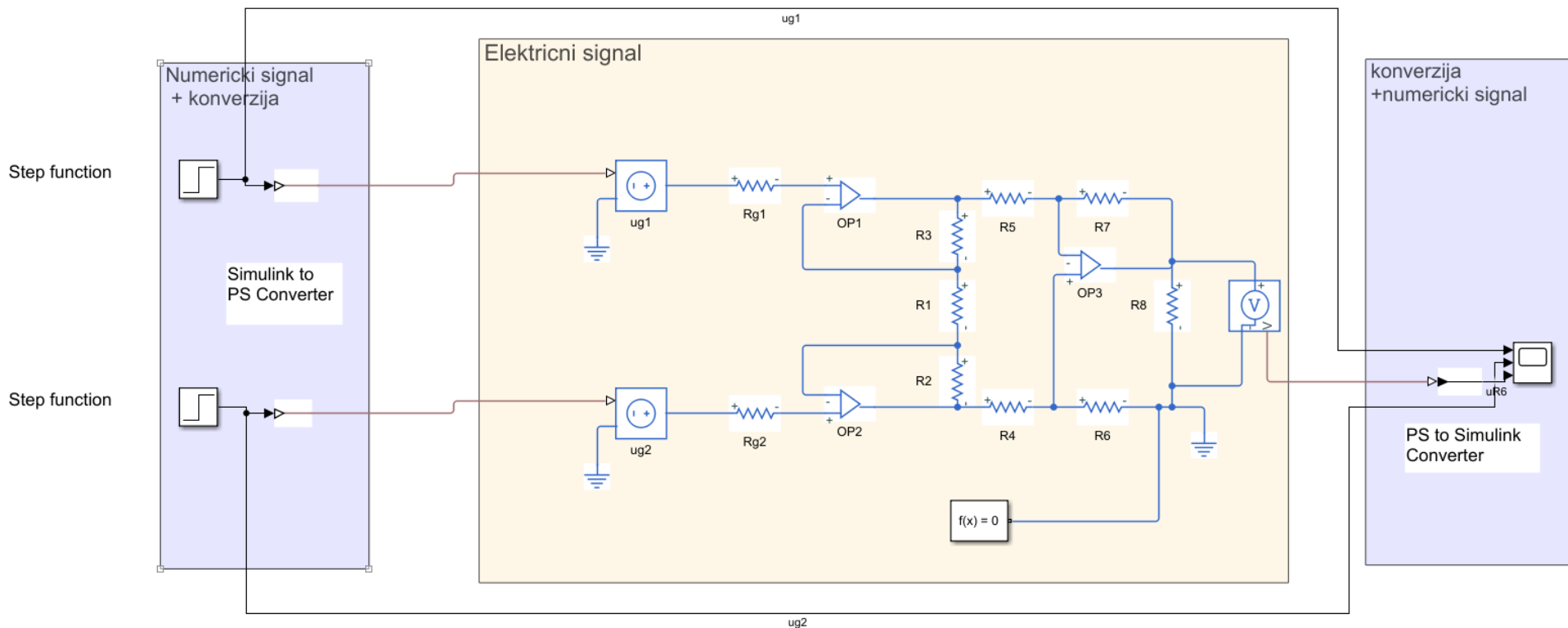
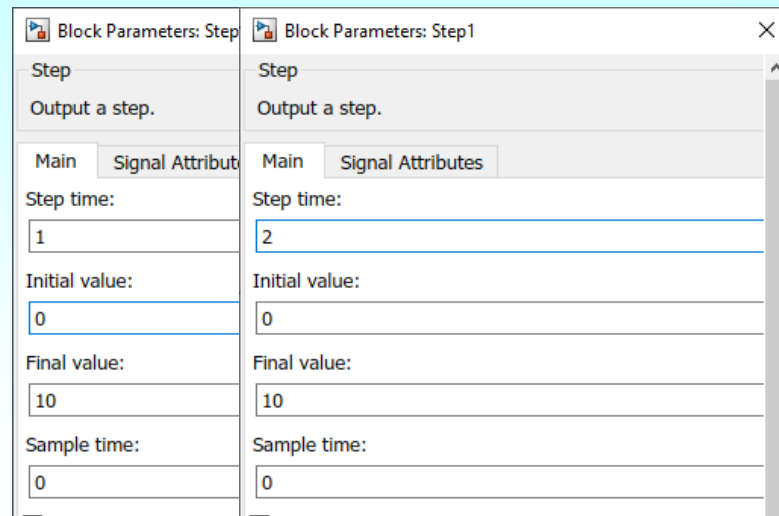


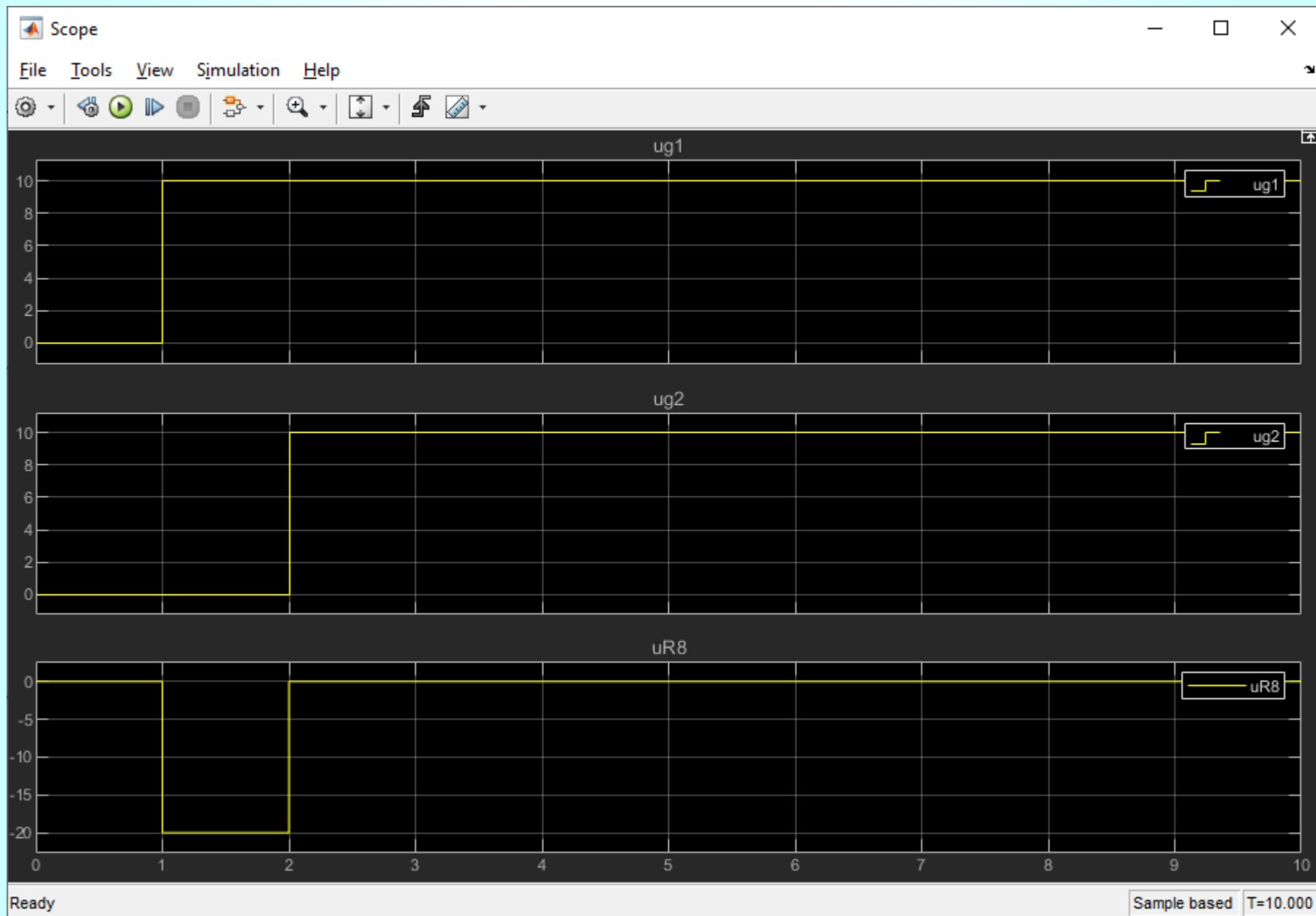
MATLAB: Simscape Utilities



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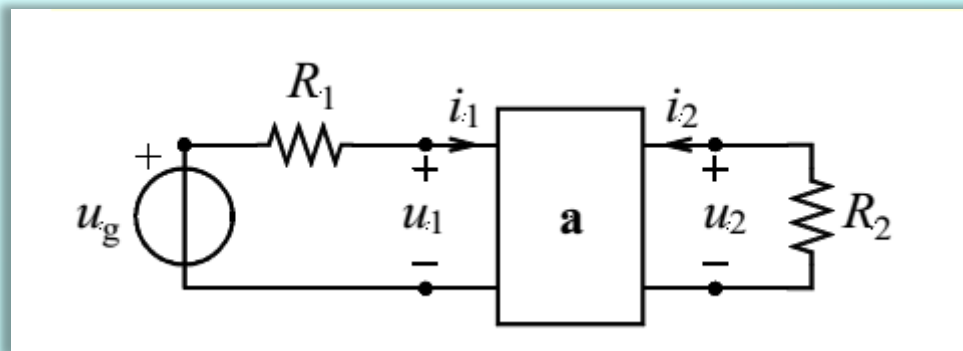
Foundation Library, Utilities, Simulink





ABCD параметри мреже

- Отпорности отпорника и a -параметри (ABCD-параметри) мреже са два приступа су познати.
- Одредити:
 - Количник напона $A = u_2 / u_g$
 - Улазну отпорност мреже $R_u = u_1 / i_1$
 - Струју i_1
- Сматрати да су a -параметри мреже реални и различити од нуле.



MATLAB: Symbolic Math Toolbox

```
clear variables
```

```
syms R1 R2 a11 a12 a21 a22 i1 i2 u1 u2 ug
jednacine = [ug == R1 * i1 + u1, u2 + R2 * i2 == sym(0), u1 == a11 * u2 + a12 * -i2, i1 == a21 * u2 + a22 * -i2]
```

$$\text{jednacine} = (ug = u_1 + R_1 i_1 \quad u_2 + R_2 i_2 = 0 \quad u_1 = a_{11} u_2 - a_{12} i_2 \quad i_1 = a_{21} u_2 - a_{22} i_2)$$

```
promenljive = [u1, u2, i1, i2]
```

$$\text{promenljive} = (u_1 \quad u_2 \quad i_1 \quad i_2)$$

```
assume(a11*a22-a12*a21 ~= 0 & R1>0 & R2>0 & ug~=0)
odziv = solve(jednacine, promenljive)
```

```
odziv = struct with fields:
    u1: (a12*ug + R2*a11*ug)/(a12 + R2*a11 + R1*a22 + R1*R2*a21)
    u2: (R2*ug)/(a12 + R2*a11 + R1*a22 + R1*R2*a21)
    i1: (ug*(a22 + R2*a21))/(a12 + R2*a11 + R1*a22 + R1*R2*a21)
    i2: -ug/(a12 + R2*a11 + R1*a22 + R1*R2*a21)
```

```
A = odziv.u2/ug
```

A =

$$\frac{R_2}{a_{12} + R_2 a_{11} + R_1 a_{22} + R_1 R_2 a_{21}}$$

```
Ru = simplify(odziv.u1/odziv.i1)
```

Ru =

$$\frac{a_{12} + R_2 a_{11}}{a_{22} + R_2 a_{21}}$$

```
simplify(odziv.i1)
```

ans =

$$\frac{ug (a_{22} + R_2 a_{21})}{a_{12} + R_2 a_{11} + R_1 a_{22} + R_1 R_2 a_{21}}$$